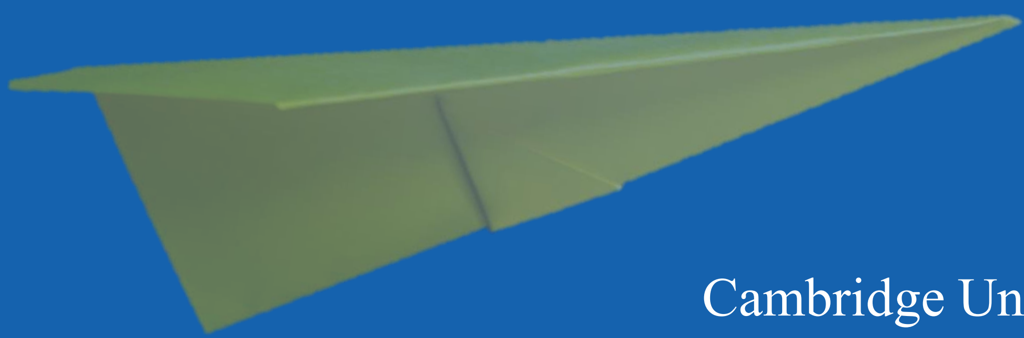
Clicker Questions

Modern Physics

by Gary Felder and Kenny Felder



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Instructions

- These questions are offered in two formats: a deck of PowerPoint slides, and a PDF file. The two files contain identical contents. There are similar files for each of the 14 chapters in the book, for a total of 28 files.
- Each question is marked as a "Quick Check" or "ConcepTest."
  - Quick Checks are questions that most students should be able to answer correctly if they have done the reading or followed the lecture. You can use them to make sure students are where you think they are before you move on.
  - ConcepTests (a term coined by Eric Mazur) are intended to stimulate debate, so you don't want to prep the class too explicitly before asking them. Ideally you want between 30% and 80% of the class to answer correctly.
- Either way, if a strong majority answers correctly, you can briefly discuss the answer and move on. If many students do not answer correctly, consider having them talk briefly in pairs or small groups and then vote again. You may be surprised at how much a minute of unguided discussion improves the hit rate.
- Each question is shown on two slides: the first shows only the question, and the second adds the correct answer.
- Some of these questions are also included in the book under "Conceptual Questions and ConcepTests," but this file contains additional questions that are not in the book.
- Some of the pages contain multiple questions with the same set of options. These questions are numbered as separate questions on the page.
- Some questions can have multiple answers. (These are all clearly marked with the phrase "Choose all that apply.") If you are using a clicker system that doesn't allow multiple responses, you can ask each part separately as a yes-or-no question.

10 Statistical Mechanics

10.1 Microstates and Macrostates

Which of the following is generally true of a macroscopic system? (Choose one.)

A. Each microstate corresponds to many possible macrostates.

B. Each macrostate corresponds to many possible microstates.

C. Neither of the above.

Which of the following is generally true of a macroscopic system? (Choose one.)

A. Each microstate corresponds to many possible macrostates.

B. Each macrostate corresponds to many possible microstates.

C. Neither of the above.

Solution: B

Which of the following is generally true of a macroscopic system? (Choose one.)

A. If you know the microstate of a system then in principle you also know the macrostate.

B. If you know the macrostate of a system then in principle you also know the microstate.

C. Neither of the above.

Which of the following is generally true of a macroscopic system? (Choose one.)

A. If you know the microstate of a system then in principle you also know the macrostate.

B. If you know the macrostate of a system then in principle you also know the microstate.

C. Neither of the above.

Solution: A

Suppose you flip a (fair) coin 6 times. Are you more likely to get (in order) HHHHHH or HHTHTT? (Choose one.)

A. HHHHHH is more likely.

B. HHTHTT is more likely.

C. They are equally likely.

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A. HHHHHH is more likely.

B. HHTHTT is more likely.

C. They are equally likely.

Solution: C

If you flip a coin twenty times, why are you more likely to get "exactly ten heads" than to get "exactly thirteen heads"? (Choose one.)

A. This is a common misconception; actually, any number of heads from 0 to 20 is equally likely.

B. If the coin is fair, a flip of "heads" on any given flip makes "tails" on the next flip more likely.

C. There are more ways to end up with exactly ten heads than there are ways to end up with thirteen heads.

If you flip a coin twenty times, why are you more likely to get "exactly ten heads" than to get "exactly thirteen heads"? (Choose one.)

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B. If the coin is fair, a flip of "heads" on any given flip makes "tails" on the next flip more likely.

C. There are more ways to end up with exactly ten heads than there are ways to end up with thirteen heads.

Solution: C

Which of the following systems follow the fundamental assumption of statistical mechanics? (Choose all that apply.)

A. A sealed, insulated canister of gas

B. A glass of cold water sitting on a counter

C. The earth

D. The universe

Which of the following systems follow the fundamental assumption of statistical mechanics? (Choose all that apply.)

A. A sealed, insulated canister of gas

B. A glass of cold water sitting on a counter

C. The earth

D. The universe

Solution: A, D

In which of the following situations will a paramagnet obey the fundamental assumption of statistical mechanics? (Choose all that apply.)

A. It is in a hermetically sealed container interacting with nothing.

B. It is in a constant, external magnetic field, but otherwise interacting with nothing.

C. It is in a constant, external magnetic field while sitting out on the counter in a hot room.

In which of the following situations will a paramagnet obey the fundamental assumption of statistical mechanics? (Choose all that apply.)

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B. It is in a constant, external magnetic field, but otherwise interacting with nothing.

C. It is in a constant, external magnetic field while sitting out on the counter in a hot room.

Solution: A and B. In A all of the microstates have equal energy. In B they do not, so only certain microstates would be accessible, but they would all be equally likely. In C you can't use that assumption because the paramagnet would exchange energy with the room.

Suppose you flip two coins, but they are weighted coins; each one is twice as likely to show up heads as tails. So there are four possible microstates of this system (HH, HT, TH, and TT), but they are not all equally probable. Is this a violation of the fundamental assumption of statistical mechanics?

Suppose you flip two coins, but they are weighted coins; each one is twice as likely to show up heads as tails. So there are four possible microstates of this system (HH, HT, TH, and TT), but they are not all equally probable. Is this a violation of the fundamental assumption of statistical mechanics?

Solution: No. Which side a coin lands on is still a macrostate, determined by countless microscopic interactions.

10.2 Entropy and the Second Law of Thermodynamics

You have two dice, each of which can show a number from 1 to 6. What is the multiplicity of the macrostate "the total of the two dice is 3"? (Choose one.)

A. 0

B. 1

C. 2

D. 3

E. 4

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A. 0

B. 1

C. 2

D. 3

E. 4

Solution: C

What is the lowest multiplicity a system can have? (Choose one.)

A. Multiplicity can be negative.

B. 0

C. 1

D. Multiplicity is always greater than 1.

What is the lowest multiplicity a system can have? (Choose one.)

A. Multiplicity can be negative.

B. 0

C. 1

D. Multiplicity is always greater than 1.

Solution: C

A system has N oscillators, each of which can have energy 0, ε, 2ε, *etc.* What is the entropy of this system when its total energy is ε?

$A.\ N$

$B.\ \varepsilon$

$C.\ N\varepsilon$

$D.\ k_B \ln N$

$E.\ k_B \ln(N\varepsilon)$

A system has N oscillators, each of which can have energy 0, ε, 2ε, *etc.* What is the entropy of this system when its total energy is ε?

$A.\ N$

$B.\ \varepsilon$

$C.\ N\varepsilon$

$D.\ k_B \ln N$

$E.\ k_B \ln(N\varepsilon)$

Solution: D

Suppose you compare two macrostates of a system, and you find State A has a higher multiplicity than State B. Is it possible for State B to have a higher entropy than State A?

Suppose you compare two macrostates of a system, and you find State A has a higher multiplicity than State B. Is it possible for State B to have a higher entropy than State A?

Solution: No. The entropy is directly proportional to the ln of the multiplicity, and ln is an ever-increasing function.

For each of the following events, does the total entropy increase, decrease, or remain the same?

A. Two cars have a head-on collision.

B. An astronaut drops a ball on the moon. (Include only the time the ball is falling, not its collision with the ground.)

C. A pot of water on the stove boils.

D. The Earth revolves around the sun.

For each of the following events, does the total entropy increase, decrease, or remain the same?

A. Two cars have a head-on collision.

   Solution: Increases. This must be true because the event is irreversible. The macrostate "car in pristine condition" has very few microstates; the macrostate "banged-up car" has many.

B. An astronaut drops a ball on the moon. (Include only the time the ball is falling, not its collision with the ground.)

   Solution: Stays the same. The video played backwards would look like a completely plausible video of a ball moving upward and slowing down, so this is reversible.

C. A pot of water on the stove boils.

   Solution: Increases. Watch the video backwards and you'll see steam from all over the room collecting in the pot. (If you cool the room down the vapor will condense back into water, but it won't all end up back in the pot.)

D. The Earth revolves around the sun.

   Solution: Stays the same. Once again this time, the backwards video looks like a reasonable orbit.

Consider a system which—unlike macroscopic systems—has only a hundred possible microstates. 94 of these microstates correspond to Macrostate 1, and the other 6 correspond to Macrostate 2. The system changes its microstate randomly 10 times every year. Which of the following is fairly certain to be true? (Choose one.)

A. The system will start in Macrostate 1.

B. After 1000 years, the system will be in Macrostate 1.

C. Over a 1000 year period, the system will spend about 94% of its time in Macrostate 1.

D. Once the system is in Macrostate 1, it will never revert to Macrostate 2.

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Solution: C. This is guaranteed by the fundamental assumption of statistical mechanics.

Can the entropy of a system ever be negative?

Can the entropy of a system ever be negative?

Solution: No. The log only gives you a negative answer if you give it a number between 0 and 1, and the multiplicity could never be less than 1.

10.3 Temperature

The direction that heat flows between two objects depends on ... (Choose one)

A. which one has a higher energy

B. which one has a higher entropy

C. whose entropy depends more strongly on energy

D. whose temperature depends more strongly on energy

E. none of the above

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A. which one has a higher energy

B. which one has a higher entropy

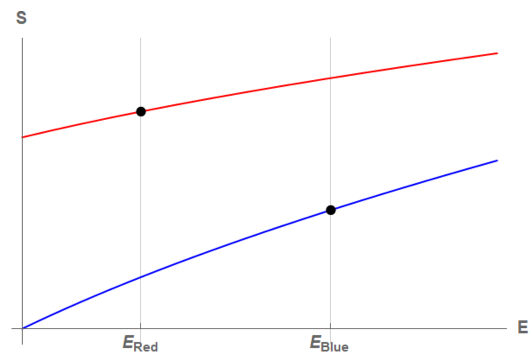
C. whose entropy depends more strongly on energy

D. whose temperature depends more strongly on energy

E. none of the above

Solution: C

The plot below shows entropy vs. energy for two systems, and shows the current energy of both systems. Which way will heat flow? (Choose one)



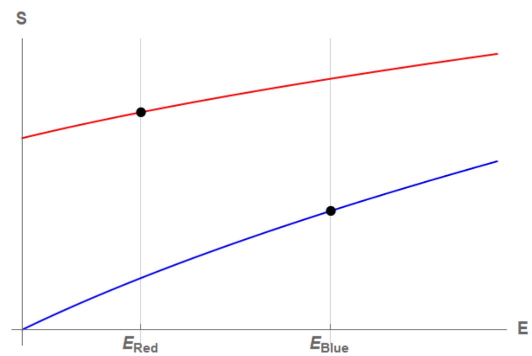
A. from the system represented by the red curve to the system represented by the blue curve

B. from the system represented by the blue curve to the system represented by the red curve

C. No heat will flow.

D. There's not enough information in the graph to tell.

The plot below shows entropy vs. energy for two systems, and shows the current energy of both systems. Which way will heat flow? (Choose one)



A. from the system represented by the red curve to the system represented by the blue curve

B. from the system represented by the blue curve to the system represented by the red curve

C. No heat will flow.

D. There's not enough information in the graph to tell.

Solution: A

For each of the following statements about two systems A and B, say whether it is A) always true, B) never true, or C) sometimes true.

1. If A has more energy than B, then A's temperature is higher.
2. If the slope of S vs. E is higher for A than for B, then A's temperature is higher.
3. If A has a higher temperature than B, then A's heat capacity is higher.
4. If the slope of S vs. E is higher for A than for B, then A's heat capacity is higher.

For each of the following statements about two systems A and B, say whether it is A) always true, B) never true, or C) sometimes true.

1. If A has more energy than B, then A's temperature is higher.

   Solution: C

2. If the slope of S vs. E is higher for A than for B, then A's temperature is higher.

   Solution: B

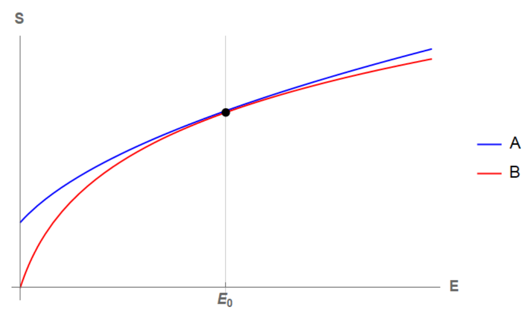
3. If A has a higher temperature than B, then A's heat capacity is higher.

   Solution: C

4. If the slope of S vs. E is higher for A than for B, then A's heat capacity is higher.

   Solution: C

The figure shows entropy as a function of energy for two systems, both of which are at energy E_0.



1. Which system has a higher temperature? (Choose one.)
   A. A
   B. B
   C. They have the same temperature.
   D. There's not enough information in the graph to tell.
2. Which system has a higher heat capacity? (Choose one.)
   A. A
   B. B
   C. They have the same heat capacity.
   D. There's not enough information in the graph to tell.

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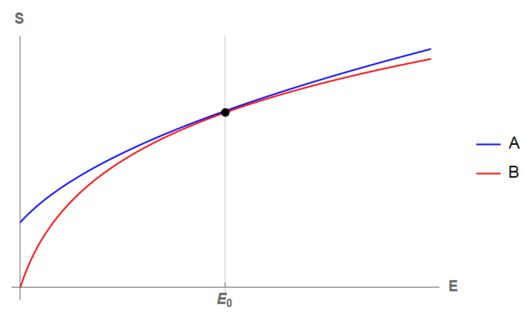

1. Which system has a higher temperature? (Choose one.)

   A. A

   B. B

   C. They have the same temperature.

   D. There's not enough information in the graph to tell.

Solution: C. You can see that the two curves are tangent at E_0, which means they have the same slope and thus the same temperature at that energy.

(Next answer on next page)

2. Which system has a higher heat capacity? (Choose one.) Explain.

   A. A

   B. B

   C. They have the same heat capacity.

   D. There's not enough information in the graph to tell.

Solution: B (red graph)

Imaging starting at $E = E_0$ on the red graph (System B) and then increasing the energy, taking a step to the right along the graph.

- The slope dS/dE decreases. (Be careful: dS/dE is a positive quantity throughout. But as the energy increases, it becomes *less* positive.)
- That means the temperature $T = 1/(dS/dE)$ increases.
- Since increasing the energy causes the temperature to increase, the derivative dT/dE is positive.
- Heat capacity is the inverse of that derivative, $C = dE/dT$, so that's also positive.

All makes sense, right? We expect both temperature and heat capacity to always be positive quantities. If you step through that same process along the blue graph (System A) you would write all the same sentences, but *less* so. Increasing the energy causes a smaller decrease in dS/dE, which causes a smaller increase in temperature, which implies a smaller derivative dT/dE. So the red graph has a higher heat capacity.

True or false? The definition of heat capacity, $C = dE/dT$, is only valid when temperature is measured in Kelvin or another scale that starts at absolute zero.

True or false? The definition of heat capacity, $C = dE/dT$, is only valid when temperature is measured in Kelvin or another scale that starts at absolute zero.

Solution: False. Changing from Kelvin to Celsius moves the entire graph E vs T graph horizontally, but it doesn't change its slope. Put another way, a formula that multiplies or divides by T will only work in an absolute temperature scale like Kelvin. But a difference in temperature dT is equally valid in any temperature scale.

10.4 The Boltzmann Distribution

A paramagnet in an external magnetic field feels a force that makes one orientation higher energy than the other. Suppose the field is pointing such that an up atom has more energy than a down atom. If the paramagnet is in contact with a reservoir, is it more likely to be in a microstate with most of the atoms pointing up or a microstate with most of them pointing down? (Choose one.)

A. The microstate with most of them pointing up is more likely.

B. The microstate with most of them pointing down is more likely.

C. The two microstates are equally likely.

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A. The microstate with most of them pointing up is more likely.

B. The microstate with most of them pointing down is more likely.

C. The two microstates are equally likely.

Solution: B

A system has many different microstates, but only two energy levels: every microstate has energy E_1 or E_2, where $E_2 > E_1$. Microstate M_1 has energy E_1, and microstate M_2 has energy E_2. Which of the following is definitely true? (Choose one.)

A. The probability of finding the system in microstate M_1 is greater than the probability of finding the system in microstate M_2.

B. The probability of finding the system with energy E_1 is greater than the probability of finding the system with energy E_2.

C. Both of those statements are definitely true.

D. Neither of those statements is definitely true.

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C. Both of those statements are definitely true.

D. Neither of those statements is definitely true.

Solution: A

System S has many particles. Each individual particle obeys the Boltzmann distribution, with the rest of the system acting as a reservoir. Which of the following best describes the behavior of System S as you lower its temperature?

A. More and more particles are found in low-energy states.

B. More and more particles are found in high-energy states.

C. All states become equally likely.

D. Changing the temperature does not change the distribution of states.

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Solution: A

System S has many particles. Each individual particle obeys the Boltzmann distribution, with the rest of the system acting as a reservoir. Which of the following best describes the behavior of System S as $T \to \infty$?

A. It approaches a state in which all particles are in the lowest-energy states possible.

B. It approaches a state in which all particles are in the highest-energy states possible.

C. All states become equally likely.

D. Changing the temperature does not change the distribution of states.

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C. All states become equally likely.

D. Changing the temperature does not change the distribution of states.

Solution: C

A hydrogen atom has two ground states (one with each possible electron spin) and eight first excited states. Consider a hydrogen atom floating in a room full of other hydrogen gas (a reservoir).

1. Is there any temperature at which one of the excited states would be more likely to occur than one of the ground states?
2. Is there any temperature at which it would be more likely for the hydrogen atom to be in a first excited state than for it to be in a ground state?

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1. Is there any temperature at which one of the excited states would be more likely to occur than one of the ground states?

   Solution: No

2. Is there any temperature at which it would be more likely for the hydrogen atom to be in a first excited state than for it to be in a ground state?

   Solution: Yes

A hot glass of water and a cold glass of water are sitting in the same room, which is at temperature T. Which of the following describes the probabilities of their microstates? (Choose one.)

A. Because they are both in contact with the same reservoir, we know from the Boltzmann distribution that the microstates of the hot glass all have the same probabilities as the corresponding microstates of the cold glass.

B. The probability distribution for the hot glass is more highly weighted towards high energy states than the probability distribution for the cold glass, thus indicating that one or both them don't obey the Boltzmann distribution with temperature T.

C. They both have the same probability distribution, but the fact that we see that one glass is hotter than the other tells us that the hot glass randomly landed in a higher energy microstate than the cold glass.

D. It is meaningless to talk about probabilities of microstates until the glasses have reached equilibrium with the surroundings.

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C. They both have the same probability distribution, but the fact that we see that one glass is hotter than the other tells us that the hot glass randomly landed in a higher energy microstate than the cold glass.

D. It is meaningless to talk about probabilities of microstates until the glasses have reached equilibrium with the surroundings.

Solution: B

A paramagnet consists of N atoms, each of which can have energy 0 or ϵ. In the limit $\lim_{T\to\infty}$, how many of the atoms are in their excited state? (Choose one.)

A. 0

B. Less than $N/2$ but more than 0

$C. N/2$

D. More than $N/2$ but less than N

$E. N$

A paramagnet consists of N atoms, each of which can have energy 0 or ϵ. In the limit $\lim_{T\to\infty}$, how many of the atoms are in their excited state? (Choose one.)

A. 0

B. Less than $N/2$ but more than 0

C. $N/2$

D. More than $N/2$ but less than N

E. N

Solution: C

At any temperature each atom is more likely to be in the ground state than in the excited state by a factor of $e^{\epsilon/(k_B T)}$. As $T \to \infty$ that factor approaches 1, so the two states are equally likely.

More generally, the Boltzmann distribution says that all microstates for any system become equally likely in the limit $T \to \infty$. (You might want to think about how you could reach the same conclusion applying the Boltzmann distribution to the entire paramagnet instead of a single atom.)

10.5 Some Applications of the Boltzmann Distribution

A typical room temperature is about 300 K. The core of the Earth is at about 6000 K, about 20 times hotter than the surface. A typical atom in the core of the Earth has… (Choose one.)

A. about $\sqrt{20}$ times as much energy as a typical atom on the Earth's surface.

B. about 20 times as much energy as a typical atom on the Earth's surface.

C. about 400 times as much energy as a typical atom on the Earth's surface.

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B. about 20 times as much energy as a typical atom on the Earth's surface.

C. about 400 times as much energy as a typical atom on the Earth's surface.

Solution: B

Which of the following conditions might lead the average energy of a system to be very far from $k_B T$? (Choose all that apply.)

A. Every state in the system has a degeneracy of exactly 32.

B. The $n = 1$ state has a degeneracy of 2, and the $n = 3$ state has a degeneracy of 10^6.

C. The excitation energy from each state to the next is greater than $10 k_B T$.

D. The excitation energy from each state to the next is less than $k_B T/100$.

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C. The excitation energy from each state to the next is greater than $10 k_B T$.

D. The excitation energy from each state to the next is less than $k_B T/100$.

Solution: B and C

If the molecules of a system can rotate that tends to… (Choose one.)

A. increase the system's heat capacity.

B. decrease the system's heat capacity.

C. have no effect on the system's heat capacity.

If the molecules of a system can rotate that tends to… (Choose one.)

A. increase the system's heat capacity.

B. decrease the system's heat capacity.

C. have no effect on the system's heat capacity.

Solution: A

The Maxwell speed distribution says which of the following about the probability distribution for molecular speeds? (Choose one.)

A. Lower speeds are always more likely than higher speeds.

B. High speeds are always more likely than lower speeds.

C. All speeds are equally likely.

D. There is a peak speed around which the molecules are most likely to be moving, and speeds lower or higher than this are less likely.

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C. All speeds are equally likely.

D. There is a peak speed around which the molecules are most likely to be moving, and speeds lower or higher than this are less likely.

Solution: D

The formula $E \sim k_B T$ can be used to approximate... (Choose all that apply.)

A. The average energy of all the particles in an isolated, macroscopic system.

B. The maximum energy of all the particles in an isolated, macroscopic system.

C. The average energy of all the particles in a small macroscopic system in contact with a reservoir.

D. The maximum energy of all the particles in a small macroscopic system in contact with a reservoir.

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A. The average energy of all the particles in an isolated, macroscopic system.

B. The maximum energy of all the particles in an isolated, macroscopic system.

C. The average energy of all the particles in a small macroscopic system in contact with a reservoir.

D. The maximum energy of all the particles in a small macroscopic system in contact with a reservoir.

Solution: A and C

The formula is for an average energy, not a maximum. It is based on the Boltzmann distribution, which applies to a small system with a reservoir. So C works, while B and D don't.

But what about A? Remember that in an isolated system, each individual particle is a small system for which the entire system acts as a reservoir, So the average energy over all the particles should still be in the ballpark of $k_B T$, where T is the temperature of the system.

A quantum simple harmonic oscillator has evenly spaced energies $E = (n + 1/2)\hbar\omega$, where ω is the angular frequency. Which would you expect to fall closer to $E = k_B T$? (Choose one.)

A. a very high frequency oscillator

B. a very low frequency oscillator

C. Frequency has no effect on the average thermal energy.

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A. a very high frequency oscillator

B. a very low frequency oscillator

C. Frequency has no effect on the average thermal energy.

Solution: B. The high frequency oscillator will have a larger spacing $\hbar\omega$ between energy states. If the frequency is high enough that $\hbar\omega \gg k_B T$ the average energy will approach zero. In the opposite limit where $\hbar\omega \ll k_B T$ the energy will approach $k_B T$.

Which would you expect to be moving faster in the air around you? (Choose one.)

A. a hydrogen molecule

B. a nitrogen molecule

C. They would on average move at the same speed.

Which would you expect to be moving faster in the air around you? (Choose one.)

A. a hydrogen molecule

B. a nitrogen molecule

C. They would on average move at the same speed.

Solution: A. On average they have the same translational kinetic energy ($(3/2)k_B T$), which means the lighter one must be moving faster.

A water molecule is made up of three atoms that are *not* all in a line with each other. Based on only that information, which of the following is the best prediction for the heat capacity, per molecule, of water vapor at room temperature? (Choose one.)

A. $(1/2)k_B$

B. k_B

C. $(3/2)k_B$

D. $(5/2)k_B$

E. $3k_B$

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C. $(3/2)k_B$

D. $(5/2)k_B$

E. $3k_B$

Solution: E

If the three molecules are not in a line, the shape can rotate in all three directions. So there are six relevant degrees of freedom here, three translational and three rotational, leading to $E \sim 3k_BT$, so the heat capacity per molecule should be $3k_B$.

10.6 Quantum Statistics

One difference between a fermion and a boson is… (Choose one.)

A. You can tell two fermions apart, but you cannot tell two bosons apart.

B. Two identical bosons can be in the same quantum state as each other, but two identical fermions cannot.

C. A boson can be in many different quantum states, but a fermion can only be in two.

D. Fermions obey classical statistics, but bosons require quantum statistics.

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Solution: B

You plug a particular energy E and a particular temperature T into the Bose-Einstein distribution, and the resulting answer is 4.5. That tells you... (Choose one.)

A. If you measure the number of particles in one particular state with energy E, and then repeat that measurement many times, the average result will be 4.5.

B. If you measure the number of particles with energy E, and then repeat that measurement many times, the average result will be 4.5.

C. The expectation value of the energy of a particle in your system is 4.5.

D. The expectation value of the number of particles in your system is 4.5.

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C. The expectation value of the energy of a particle in your system is 4.5.

D. The expectation value of the number of particles in your system is 4.5.

Solution: A

You can reasonably ignore this section, and just use the Boltzmann distribution from Section 10.4, when... (Choose all that apply.)

A. You're looking for the occupancy of a very high-energy state.

B. You're looking for the occupancy of a very low-energy state.

C. The particles in question are all identical, and cannot be distinguished from each other in any way.

D. The particles in question *can* be distinguished from each other in some way.

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B. You're looking for the occupancy of a very low-energy state.

C. The particles in question are all identical, and cannot be distinguished from each other in any way.

D. The particles in question *can* be distinguished from each other in some way.

Solution: A and D

A "degenerate Fermi gas" is… (Choose one.)

A. A collection of fermions that are all in the lowest states possible.

B. A collection of fermions that are all in the same state as each other.

C. A collection of fermions that are all in different states with the same energy.

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Solution: A

We said that for a system of identical fermions $\overline{n} = 1/3$ means that the probability of finding a particle in that state is 1/3. Can you similarly use $\overline{n} = 1/3$ to determine the probability of finding a particle in that state for a system of bosons?

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Solution: No. It works for fermions because each state is either occupied or not, end of story. For a boson, that result might mean for instance that there is a 1/100 chance of finding 300 particles there, and otherwise it will be empty. (OK, that sounds unlikely, but you get the point. The average value is a weighted average over all the possible numbers of particles in the state.)

For a given system of fermions, the Fermi energy is 0. Which of the following best describes what that fact tells us? (Choose one.)

A. If the system is in its ground state, the fermions all have energy zero.

B. If the system is in its ground state, half the fermions have energy below zero and half above.

C. If the system is in its ground state, all the fermions have energy at or below zero.

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C. If the system is in its ground state, all the fermions have energy at or below zero.

D. If the system is in its ground state, all the fermions have energy at or above zero.

Solution: C

A system of identical particles is in contact with a reservoir. The Boltzmann distribution applies to . . . (Choose one.)

A. individual particles in the system

B. the system as a whole

C. both

D. neither

A system of identical particles is in contact with a reservoir. The Boltzmann distribution applies to ... (Choose one.)

A. individual particles in the system

B. the system as a whole

C. both

D. neither

Solution: B

The Boltzmann distribution does apply to the system as a whole. We sometimes also apply the Boltzmann distribution to individual particles (with the system acting as the reservoir) but that only applies to distinguishable particles.

The Fermi-Dirac and Bose-Einstein distributions give you the expected number of particles in a given state. If you add more particles to the system, will that increase $\overline{n}$? (Assume the density of states doesn't change.)

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Solution: Yes, more particles will increase the occupancy. That comes out in the math as a different value of μ, which we remember is a normalization constant chosen by setting the sum of all states to N.

You have a bunch of non-interacting fermions in a bound system. (If you want to picture something definite you can imagine an atom, or an infinite square well.) If you increase the number of particles does the Fermi energy … (Choose one.)

A. increase

B. decrease

C. stay the same

D. increase when N is small and then decrease once N is large enough

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A. increase

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C. stay the same

D. increase when N is small and then decrease once N is large enough

Solution: A

In general it will increase. Remember that the Fermi energy is the average of the N^{th} and $(N+1)^{\text{st}}$ energies, starting from the ground. In general the 7^{th} energy level will be higher than the 6^{th} and so on up. The only exception is when you have perfectly degenerate energy levels, in which case going up through them will not change the Fermi energy.

We said in the text that ϵ_F is independent of temperature, but both μ and ϵ_F depend on N.

You have a bunch of non-interacting fermions in a bound system. (If you want to picture something definite you can imagine an atom, or an infinite square well.) If you increase the temperature does the chemical potential μ … (Choose one.)

A. increase

B. decrease

C. stay the same

D. The answer is different for different systems of fermions.

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A. increase

B. decrease

C. stay the same

D. The answer is different for different systems of fermions.

Solution: B

Remember what μ means for a collection of fermions. For energy levels below μ, the occupation number is greater than 1/2; those states are more likely to be occupied than unoccupied.

What does that look like at very low temperatures? The fermions crowd into the lowest energy states possible, but they can't all go into the lowest state (Pauli exclusion principle). A system of N fermions will occupy the lowest N states, so μ is the Fermi energy, roughly the energy of the N^{th} state.

As you raise the temperature, the probability curve flattens. Higher-energy states now have $\overline{n}$-values higher than 0, and lower-energy states now have $\overline{n}$-values less than 1.

Keep raising the temperature and eventually that N^{th} state will drop its occupation number below 1/2. At that point, μ is less than the energy of that state. Later the state below that one falls below $\overline{n} = 1/2$, and μ drops further.

When T gets sufficiently high, you have N particles evenly distributed among far more than N energy states. The occupation number of *every* state is barely above zero, so μ is actually below the ground state.

10.7 Blackbody Radiation

Blackbody A and Blackbody B are the same shape and size, but A is hotter than B. Which of the following is true? (Choose one.)

A. A emits more radiation than B, but their radiation spectra peak at the same frequency.

B. A emits the same total amount of radiation as B, and their radiation spectra peak at different frequencies.

C. The total amount of radiation they emit, and their peak emission frequencies, are both different

D. Their emission spectra are identical.

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C. The total amount of radiation they emit, and their peak emission frequencies, are both different

D. Their emission spectra are identical.

Solution: C

A “blackbody” is an object that . . . (Check all that apply.)

A. absorbs all radiation that hits it.

B. reflects all radiation that hits it.

C. emits more thermal radiation (per surface area) than any other object at the same temperature.

D. emits less thermal radiation (per surface area) than any other object at the same temperature.

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D. emits less thermal radiation (per surface area) than any other object at the same temperature.

Solution: A and C

If you integrated the Planck spectrum from 3.8×10^{14} Hz to 7.7×10^{14} Hz (the frequency range of visible light), which of the following best describes what the answer would represent? (Choose one.)

A. The number of photons of visible light in a fully enclosed cavity at equilibrium.

B. The photon density (photons per unit volume) of the visible light in a fully enclosed cavity at equilibrium.

C. The amount of energy represented by the visible light in a fully enclosed cavity at equilibrium.

D. The energy density (energy per unit volume) of the visible light in a fully enclosed cavity at equilibrium.

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C. The amount of energy represented by the visible light in a fully enclosed cavity at equilibrium.

D. The energy density (energy per unit volume) of the visible light in a fully enclosed cavity at equilibrium.

Solution: D

Blackbody A and Blackbody B are the same shape and size, but A is hotter than B. Which of the following are true? (Choose all that apply.)

A. A emits more total power than B.

B. The radiation emitted by A is, on average, higher frequency than B's.

C. At low frequencies B emits more than A.

D. At high frequencies A emits more than B.

E. A absorbs a higher percentage of the radiation hitting it than B does.

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Solution: A, B, D

1. Can an item look black and not be a very good blackbody?
2. Can an item be a (nearly perfect) blackbody and not look black?

1. Can an item look black and not be a very good blackbody?

   Solution: Yes, if it reflects a significant amount of electromagnetic radiation that is not in the visible spectrum then it isn't a blackbody, but it could still look black if it is not reflecting or emitting much visible light.

2. Can an item be a (nearly perfect) blackbody and not look black?

   Solution: Yes, like the sun, which doesn't reflect but does emit.

10.8 Bose-Einstein Condensation

The term “condense” in this section refers to:

A. A collection of bosons all dropping into their ground states.

B. A collection of bosons all occupying different energy states, like fermions.

C. A collection of bosons occupying a microstate that maximizes the entropy of the system.

D. A gas changing state into a liquid.

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D. A gas changing state into a liquid.

Solution: A

At a low nonzero temperature, which of the following systems will have a higher percentage of particles in their ground states? (Choose one.)

A. Identical bosons

B. Identical fermions

C. Distinguishable particles

D. All three will be the same at any given temperature

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A. Identical bosons

B. Identical fermions

C. Distinguishable particles

D. All three will be the same at any given temperature

Solution: A

Why don't fermions form Bose-Einstein condensates? (Choose one.)

A. They repel each other electrically.

B. The degeneracy at high energies is so high it makes it likely for many fermions to be in higher energy states.

C. They can't exist in the same state as each other.

D. They do the same thing, but we don't use that name for it.

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A. They repel each other electrically.

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C. They can't exist in the same state as each other.

D. They do the same thing, but we don't use that name for it.

Solution: C

The text describes a system of N particles, each of which has two energy levels: 0 and ϵ. Let n_0 be the occupation number of the ground state. Answer Parts 1–4 of this question with the following choices:

A. 0

B. 1

C. $N/2$

D. N

E. None of the above

1. What is $\lim_{T \to 0^+} n_0$ for a system of distinguishable particles?
2. What is $\lim_{T \to 0^+} n_0$ for a system of identical bosons?
3. What is $\lim_{T \to \infty} n_0$ for a system of distinguishable particles?
4. What is $\lim_{T \to \infty} n_0$ for a system of identical bosons?
5. For temperatures between these two limits, is n_0 always bigger for the system of bosons than for the distinguishable particles, always bigger for the distinguishable particles, or neither?

1. What is $\lim_{T \to 0^+} n_0$ for a system of distinguishable particles?

   Solution: N because the more energy you remove from the system, the more particles fall into their ground states.

2. What is $\lim_{T \to 0^+} n_0$ for a system of identical bosons?

   Solution: N for the same reason

3. What is $\lim_{T \to \infty} n_0$ for a system of distinguishable particles?

   Solution: $N/2$. There are two states available to any given particle in this system: the $E = 0$ state, and the $E = \epsilon$ state. In the limit as $T \to \infty$ their Boltzmann factors $e^{-E/(k_B T)}$ both approach 1, meaning the two states are equally likely, so you should expect to find half the particles in each state.

4. What is $\lim_{T \to \infty} n_0$ for a system of identical bosons?

   Solution: $N/2$. In this case we do not apply the Boltzmann distribution to one particle, but to the entire system. The system has one state with energy 0, one state with energy ϵ, one state with energy 2ϵ, and so on up to $N\epsilon$. In the limit as $T \to \infty$ their Boltzmann factors $e^{-E/(k_B T)}$ all approach 1, so all states are equally likely.

   So if you perform many measurements of the system, how many particles on average will you find in the ground state? If the whole system has energy 0, you will find all of them (N); if the whole system has energy ϵ, you will find $N - 1$ of them; and so on, up until the highest-energy state of the system where you will find no particles at all in their ground states. Since all of these results are equally likely you can just average them to get $n_0 = N/2$.

5. For temperatures between these two limits, is n_0 always bigger for the system of bosons than for the distinguishable particles, always bigger for the distinguishable particles, or neither?

   Solution: Bigger for the bosons. As you go up in energy levels the Boltzmann distribution—the probability of any given microstate—looks the same for both systems. But the system of bosons has one microstate at each energy level. For the distinguishable particle system, higher energy levels are represented by more microstates, and are therefore more probable.