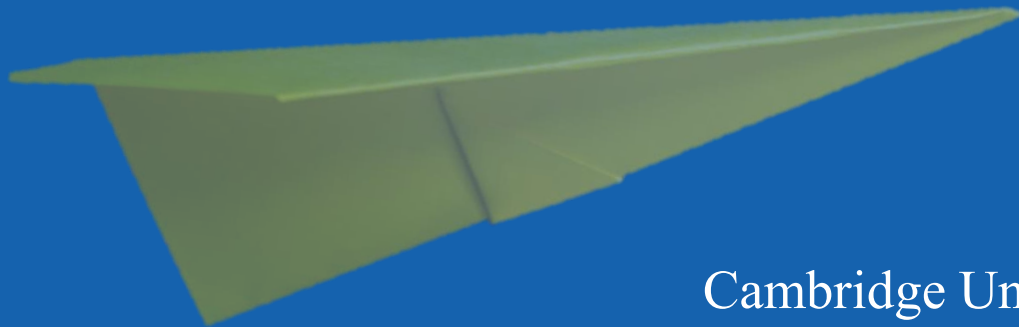


# Clicker Questions

## *Modern Physics*

by Gary Felder and Kenny Felder



Cambridge University Press

[cambridge.org/core/resources/felder-modernphysics/](http://cambridge.org/core/resources/felder-modernphysics/)  
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# Instructions

- . These questions are offered in two formats: a deck of PowerPoint slides, and a PDF file. The two files contain identical contents. There are similar files for each of the 14 chapters in the book, for a total of 28 files.
- . Each question is marked as a “Quick Check” or “ConcepTest.”
  - Quick Checks are questions that most students should be able to answer correctly if they have done the reading or followed the lecture. You can use them to make sure students are where you think they are before you move on.
  - ConcepTests (a term coined by Eric Mazur) are intended to stimulate debate, so you don’t want to prep the class too explicitly before asking them. Ideally you want between 30% and 80% of the class to answer correctly.
- . Either way, if a strong majority answers correctly, you can briefly discuss the answer and move on. If many students do not answer correctly, consider having them talk briefly in pairs or small groups and then vote again. You may be surprised at how much a minute of unguided discussion improves the hit rate.
- . Each question is shown on two slides: the first shows only the question, and the second adds the correct answer.
- . Some of these questions are also included in the book under “Conceptual Questions and ConcepTests,” but this file contains additional questions that are not in the book.
- . Some of the pages contain multiple questions with the same set of options. These questions are numbered as separate questions on the page.
- . Some questions can have multiple answers. (These are all clearly marked with the phrase “Choose all that apply.”) If you are using a clicker system that doesn’t allow multiple responses, you can ask each part separately as a yes-or-no question.

# 4

## The Quantum Revolution II: Matter and Wavefunctions

## **4.1 Atomic Spectra and the Bohr Model**

Which of the following did Rutherford's gold foil experiment show that the plum pudding model didn't predict? (Choose one.)

- A. The positive charge in an atom is much heavier than the negative charge.
- B. Atoms are smaller than was previously believed.
- C. The positive charge in an atom is evenly spread out.
- D. The positive charge in an atom is concentrated in a small region.

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- D. The positive charge in an atom is concentrated in a small region.

**Solution:** D

Which of the following are quantized in the Bohr model? (Choose all that apply.)

- A. The radius of the electron's orbit.
- B. The electron's angular momentum.
- C. The electron's energy.
- D. The force between the electron and the nucleus.

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**Solution:** All of them



What does the Franck-Hertz experiment demonstrate? (Choose all that apply.)

- A. Atoms have quantized energy levels.
- B. Electrons can knock atoms into higher energy levels.
- C. Mercury has two energy levels separated by 4.9 eV.
- D. Mercury has infinitely many energy levels separated by 4.9 eV each.

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- D. Mercury has infinitely many energy levels separated by 4.9 eV each.

**Solution:** A, B, C

Consider doubly-ionized lithium, an ion with three protons and one electron. The highest frequency spectral line of this ion would be ...

- A. higher than the highest frequency for hydrogen.
- B. equal to the highest frequency for hydrogen.
- C. lower than the highest frequency for hydrogen.

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- B. equal to the highest frequency for hydrogen.
- C. lower than the highest frequency for hydrogen.

**Solution:** A

Which of the following are true of the spectral lines of hydrogen?  
(Choose all that apply.)

- A. There are infinitely many emission lines.
- B. There is a highest frequency at which hydrogen can emit.
- C. There is a lowest frequency at which hydrogen can emit.
- D. All of the frequencies of absorption lines are also frequencies of emission lines.

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- D. All of the frequencies of absorption lines are also frequencies of emission lines.

**Solution:** A, B, D

In the Franck-Hertz experiment the current dropped when the voltage went above 4.9 V because of electrons knocking mercury atoms into the energy state 4.9 eV higher than their ground state. Why did the current drop again when the voltage reached 9.8 eV? (Choose one.)

- A. Some electrons excited mercury atoms to a level 9.8 eV above the ground state.
- B. Some electrons collided with two mercury atoms.
- C. Some mercury atoms were struck by two electrons.

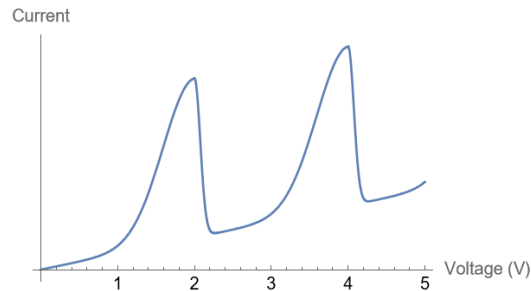
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- C. Some mercury atoms were struck by two electrons.

**Solution:** B



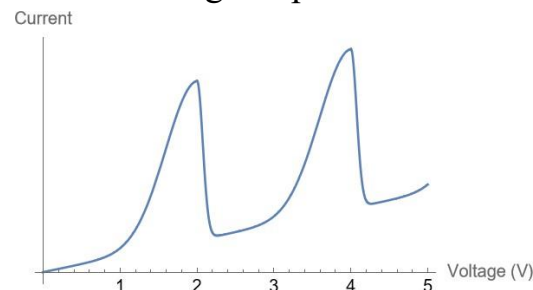
Suppose you perform a Franck-Hertz experiment on a gas of the element maduponium and you measure the following output current as a function of voltage.



Which of the following would you conclude? (Choose one.)

- A. Maduponium has a 2 eV transition (meaning it has two electron states separated by 2 eV).
- B. Maduponium has a 4 eV transition.
- C. Maduponium has both a 2 eV transition and a 4 eV transition.
- D. You cannot conclude that maduponium has either of these transitions.

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- B. Maduponium has a 4 eV transition.
- C. Maduponium has both a 2 eV transition and a 4 eV transition.
- D. You cannot conclude that maduponium has either of these transitions.

**Solution:** A. You can infer that maduponium has a 2 eV transition because the current drops suddenly at 2 V, which indicates that when electrons have 2 eV of energy they can excite a transition in maduponium atoms. You cannot infer that maduponium has a 4 eV transition. The second drop in current at 4 V indicates that each electron can excite *two* maduponium atoms, with 2 eV each.

## 4.2 Matter Waves

Each case below describes a moving electron and proton. For each one choose whether A) the electron has a larger de Broglie wavelength, B) the proton has a larger de Broglie wavelength, or C) the two have equal de Broglie wavelengths.

1. The electron and proton are moving at the same speed.
2. The electron and proton have the same momentum.

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1. The electron and proton are moving at the same speed.

**Solution:** A

2. The electron and proton have the same momentum.

**Solution:** C

Which of the following is evidence that electrons have associated matter waves? (Choose all that apply.)

- A. Electrons always show up at one place when you measure them.
- B. Electrons show interference patterns in a double slit experiment.
- C. When an electron scatters off an atom it can emit a photon.

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- C. When an electron scatters off an atom it can emit a photon.

**Solution:** B

You do a double slit experiment with electrons and observe a region near the middle where very few electrons ever hit. If you cover up one of the slits, which of the following will occur?

- A. Even fewer electrons will hit in that region.
- B. The same number of electrons will hit that region.
- C. More electrons will start hitting that region.



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- B. The same number of electrons will hit that region.
- C. More electrons will start hitting that region.

**Solution:** C

An electron and proton have the same kinetic energy. Which one of the following is true about them? Explain how you know.

- A. The electron has a larger de Broglie wavelength
- B. The proton has a larger de Broglie wavelength
- C. The two have equal de Broglie wavelengths.

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**Solution:** A. There are several ways to see this. One is to start from  $p = mv$  and  $K = (1/2)mv^2$  to derive  $K = p^2/(2m)$ . So if they have equal kinetic energies then the heavier one has the bigger momentum, and thus the smaller one has the larger de Broglie wavelength.

An electron and a photon have equal total energy (including the mass energy of the electron). Which one has a bigger de Broglie wavelength: A) the electron, B) the photon, or C) they are the same?

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**Solution:** A

For the photon  $E = pc$  while for the electron  $E = \sqrt{p^2 c^2 + m^2 c^4}$ , so if the two have equal energy the electron must have less momentum. (If that's not obvious turn it around; equal momenta would clearly give the electron a higher energy.) Thus the electron has a bigger de Broglie wavelength.

Does an electron's de Broglie wavelength depend on the observer's reference frame?

Does an electron's de Broglie wavelength depend on the observer's reference frame?

**Solution:** Yes, because wavelength depends on momentum, which depends on velocity, which depends on reference frame.

## **4.3 Wavefunctions and Position Probabilities**



You are most likely to find a particle in a region where its wavefunction equals ... (Choose one.)

A. 2

B. 0

C. -5

You are most likely to find a particle in a region where its wavefunction equals ... (Choose one.)

A. 2

B. 0

C.  $-5$

**Solution:** C

A probability distribution is only defined at  $x = 1$ ,  $x = 2$ , and  $x = 3$ . The probabilities of  $x = 1$  and  $x = 2$  are both  $1/5$ . If the distribution is properly normalized, then the probability of  $x = 3$  is... (Choose one.)

A.  $1/5$

B.  $2/5$

C.  $3/5$

D. There is not enough information here to determine  $P(3)$ .

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A.  $1/5$

B.  $2/5$

C.  $3/5$

D. There is not enough information here to determine  $P(3)$ .

**Solution:** C

The function  $\psi(x)$  is continuous over the entire real number line, and is properly normalized. Which of the following must be true? (Choose all that apply.)

A.  $\int_{-\infty}^{\infty} \psi(x) dx = 1$

B.  $\int_{-\infty}^{\infty} |\psi|^2(x) dx = 1$

C.  $|\psi|^2(10)$  gives the probability of finding the particle at  $x = 10$ .

D.  $\int_{10}^{10.1} |\psi|^2(x) dx$  gives the probability of finding the particle somewhere between  $x = 10$  and  $x = 10.1$ .

E. The probability of finding the particle at  $x = 10$  is zero.

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E. The probability of finding the particle at  $x = 10$  is zero.

**Solution:** B, D, and E

Which of the following could be the wavefunction of a particle? (The letters  $A$ ,  $B$ , and  $C$  represent constants. Choose all that apply.)

A.  $\psi(x) = Ax^3$

B.  $\psi(x) = B \sin x$

C.  $\psi(x) = C(x^2 - 1)$  from  $x = -1$  to  $x = 1$  and 0 everywhere else

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C.  $\psi(x) = C(x^2 - 1)$  from  $x = -1$  to  $x = 1$  and 0 everywhere else

**Solution:** C only



If a particle has  $2/3$  probability of being at  $x = 1$ , and  $1/6$  probability (each) of being at  $x = 2$  or  $x = 3$ , is  $\langle x \rangle \dots$  (Choose one. This should require no calculations, and less than thirty seconds of thought.)

A.  $< 1$

B.  $1$

C. between  $1$  and  $2$

D.  $2$

E.  $> 2$

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A.  $< 1$

B.  $1$

C. between  $1$  and  $2$

D.  $2$

E.  $> 2$

**Solution:** C

A particle has the wavefunction  $\psi(x) = A \sin(\pi x/L)$  from  $x = 0$  to  $x = L$  and 0 everywhere else. What is  $\langle x \rangle$  for this particle?

*Hint:* Sketch a graph. (Choose one.)

A. 0

B.  $L/4$

C.  $L/2$

D.  $L$

E. It's not defined for this wavefunction.

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C.  $L/2$

D.  $L$

E. It's not defined for this wavefunction.

**Solution:** C

A particle has the wavefunction  $\psi = Ae^{-(x/L)^2}$ . You measure it and find it very close to  $x = 2L$ . Then you measure its position again. The second time are you ... (Choose one.)

- A. more likely to find it near  $x = 2L$  than you had been the first time.
- B. less likely to find it near  $x = 2L$  than you had been the first time.
- C. just as likely to find it near  $x = 2L$  as you had been the first time.

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- C. just as likely to find it near  $x = 2L$  as you had been the first time.

**Solution:** A (more likely), because the first measurement collapsed the wavefunction.

1. In a discrete universe with a finite number of points, would it be possible to have  $|\psi|^2 = 3$  at one of those points?
2. In a continuous universe, would it be possible to have  $|\psi|^2 = 3$  at one  $x$ -value?
3. Is it possible to have a normalized function in a discrete universe with an infinite amount of points (such as “all positive integers”)?

1. In a discrete universe with a finite number of points, would it be possible to have  $|\psi|^2 = 3$  at one of those points? Why or why not?

**Solution:** No, a probability can never be greater than 1.

2. In a continuous universe, would it be possible to have  $|\psi|^2 = 3$  at one  $x$ -value? Why or why not?

**Solution:** Yes, as long as it was that large in a region smaller than  $\Delta x = 1/3$  the probability in that region would still be less than 1.

3. Is it possible to have a normalized function in a discrete universe with an infinite amount of points (such as “all positive integers”)?

**Solution:** Yes, an infinite series of numbers can converge. For example, if  $|\psi|^2$  equaled  $1/2$ , then  $1/4$ ,  $1/8$ , and so on, it would all add up to 1.



A particle has the wavefunction  $\psi(x) = A \sin(\pi x/L)$  from  $x = 0$  to  $x = L$ . Compare the probability of finding the particle in the range  $0 < x < L/2$  to the probability of finding it in the range  $L/2 < x < L$ . Choose one of the following. You do not need to do any calculations to answer this question, but you should briefly explain your reasoning.

- A. It's more likely to be found in  $0 < x < L/2$ .
- B. It's more likely to be found in  $L/2 < x < L$ .
- C. The two are equally likely.

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A. It's more likely to be found in  $0 < x < L/2$ .

B. It's more likely to be found in  $L/2 < x < L$ .

C. The two are equally likely.

**Solution:** C. If you graph the function it looks the same in both regions (just mirror-imaged), so the probabilities are the same.

Can  $\langle x \rangle$  for a particle be at a point where  $\psi = 0$ ? If so, give an example. If not, say why not.

Can  $\langle x \rangle$  for a particle be at a point where  $\psi = 0$ ? If so, give an example. If not, say why not.

**Solution:** Yes. Imagine that  $\psi$  consists of two identical humps, with  $\psi = 0$  right in the middle. So you will end up left of the middle, and right of the middle, with equal probabilities, and average out in the middle where  $\psi = 0$ .

The calculation  $\int_a^b |\psi(x)|^2 dx$  gives the probability of measuring a particle with... (Choose one, and explain your answer.)

A.  $a < x < b$

B.  $a \leq x \leq b$

C. Both of those, because they are both the same.

D. Neither of those. (If you choose this, say what it does calculate instead!)

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C. Both of those, because they are both the same.

D. Neither of those. (If you choose this, say what it does calculate instead!)

**Solution:** C. These are the same since the probability of finding the particle exactly at a particular point, such as  $x = a$  or  $x = b$ , is always zero.

## 4.4 The Heisenberg Uncertainty Principle

A wavefunction with low position uncertainty looks like . . . (Choose one.)

A. a narrow spike.

B. a sine wave.

C. a wide bump.



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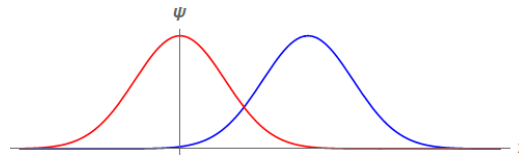
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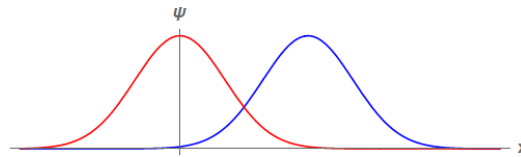
**Solution:** A

The red and blue functions below represent two wavefunctions, identical in shape but shifted along the  $x$  axis from each other. Which one has a higher position uncertainty?



- A. Red
- B. Blue
- C. They are equal.

The red and blue functions below represent two wavefunctions, identical in shape but shifted along the  $x$  axis from each other. Which one has a higher position uncertainty?



- A. Red
- B. Blue
- C. They are equal.

**Solution:** C

Knowing the uncertainty in a particle's position tells you . . . (Choose one.)

- A. what its momentum uncertainty is.
- B. an upper bound on its momentum uncertainty.
- C. a lower bound on its momentum uncertainty.

Knowing the uncertainty in a particle's position tells you . . . (Choose one.)

- A. what its momentum uncertainty is.
- B. an upper bound on its momentum uncertainty.
- C. a lower bound on its momentum uncertainty.

**Solution:** C

Which of the following are true about the uncertainty principle? (Choose all that apply.)

- A. It's impossible to know with certainty the position and momentum of a particle.
- B. Particles with large values of position have small momenta and vice-versa.
- C. It is impossible to have a particle with very uncertain position and momentum.
- D. The uncertainty relationship between position and momentum is just one example; many other uncertainty relationships exist.
- E. As our measurement technology improves, the maximum uncertainty is gradually reducing.
- F. The more energy a particle has, the faster its other measurable quantities must change.

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A. It's impossible to know with certainty the position and momentum of a particle.

**Solution:** T

B. Particles with large values of position have small momenta and vice-versa.

**Solution:** F

C. It is impossible to have a particle with very uncertain position and momentum.

**Solution:** F

D. The uncertainty relationship between position and momentum is just one example; many other uncertainty relationships exist.

**Solution:** T

E. As our measurement technology improves, the maximum uncertainty is gradually reducing.

**Solution:** F

F. The more energy a particle has, the faster its other measurable quantities must change.

**Solution:** F

Economists measure the “wealth inequality” of a nation or region. If they find a large wealth inequality, that means... (Choose one.)

- A. The average income is low.
- B. The average income is high.
- C. The standard deviation of income is low.
- D. The standard deviation of income is high.
- E. None of the above.



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- E. None of the above.

**Solution:** D

Is it possible for two wavefunctions to have the same  $\Delta x$  but different values of  $\Delta p$ ?

Is it possible for two wavefunctions to have the same  $\Delta x$  but different values of  $\Delta p$ ?

**Solution:** Yes. For instance, one might have  $\Delta p = \hbar/(2\Delta x)$  (the minimum allowed), and the other might have more. Any values greater than or equal to that  $\Delta p$  are allowed.

Which of the following is the best estimate of the uncertainty of a die roll? (Choose one.) Explain your answer.

A. 0

B. 1.5

C. 6

D.  $\infty$

Which of the following is the best estimate of the uncertainty of a die roll? (Choose one.) Explain your answer.

- A. 0
- B. 1.5
- C. 6
- D.  $\infty$

**Solution:** B

The best guess of these options is 1.5. The average die roll is 3.5. Every die roll ranges from being 0.5 to 2.5 away from the mean, so the *average* die roll is 1.5 away.

Can an object's position uncertainty ever be negative? Explain.

Can an object's position uncertainty ever be negative? Explain.

**Solution:** No. Uncertainty is (roughly speaking) an average distance from the mean. Some measurements are above the mean, and some below, but each has a positive *distance* from the mean. This is reflected in the fact that these distances are squared in the formula for standard deviation.

Is it possible to simultaneously know  $x$  and  $p_y$  with arbitrarily high precision?



Is it possible to simultaneously know  $x$  and  $p_y$  with arbitrarily high precision?

**Solution:** Yes; there is no uncertainty relationship between them.

In the limit in which a measurement of a particle's position approaches zero uncertainty, what happens to our knowledge of the particle's *velocity*? (Choose one.)

- A. As  $\Delta x \rightarrow 0$ , that causes  $\Delta v$  to approach infinity. This means that we may sometimes have to say “This particle may be going faster than the speed of light.” (If you choose this, explain why this is allowed by relativity.)
- B. Because  $v$  must always be between  $-c$  and  $c$ , the uncertainty  $\Delta v$  can approach  $c$  at the most. This puts fundamental limits on the accuracy with which we can measure position.
- C. Because  $v$  must always be between  $-c$  and  $c$ , the uncertainty  $\Delta v$  can approach  $c$  at the most. But this does *not* limit the accuracy with which we can measure position. (If you choose this, explain why it does not violate the uncertainty principle.)

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- C. Because  $v$  must always be between  $-c$  and  $c$ , the uncertainty  $\Delta v$  can approach  $c$  at the most. But this does *not* limit the accuracy with which we can measure position. (If you choose this, explain why it does not violate the uncertainty principle.)

**Solution:** C. As  $\Delta x \rightarrow 0$ ,  $\Delta p \rightarrow \infty$ . But as  $v$  approaches  $c$  the momentum approaches  $\infty$ . So you can have an arbitrarily large spread in momentum while still obeying  $v < c$ .