

Modern Physics

Answers to Odd-Numbered Problems

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CAMBRIDGE
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Relativity I: Time, Space, and Motion

1.1.3 Questions and Problems: Galilean Relativity

13. (a) $25 \text{ mph } \hat{j}$
(b) 0
(c) $50 \text{ mph } \hat{j}$
(d) $-25 \text{ mph } \hat{i} + 25 \text{ mph } \hat{j}$
15. $t' = t$, $x' = x - (10\sqrt{3} \text{ m/s})t$, $y' = y - (10 \text{ m/s})t$, $v'_x = v_x - (10\sqrt{3} \text{ m/s})$, $v'_y = v_y - (10 \text{ m/s})$,
 $a'_x = a_x$, $a'_y = a_y$
17. $t' = t$, $a'_x = a_x$, $a'_y = a_y$, $v'_x = v_x - u \cos \phi$, $v'_y = v_y - u \sin \phi$, $x' = x - (u \cos \phi)t$,
 $y' = y - (u \sin \phi)t$
19. (a) $x' = (1/3) \sin(24t)$
(b) $x = (1/3) \sin(24t) + 8t$
21. (a) $v_{C0}t + (1/2)at^2 = x_{T0} + v_Tt$
(b) -400 mi/hr^2
23. (a) $v_x = v_y = 5/\sqrt{2}$
(b) $v_x = 10 + 5/\sqrt{2}$, $v_y = 5/\sqrt{2}$
(c) $x_b(t) = (10 + 5/\sqrt{2})t$, $y_b(t) = (5/\sqrt{2})t$
(d) $x_c(t) = (10/\sqrt{2}) + 10t$, $y_c(t) = 10/\sqrt{2}$
(e) $t = 2$
25. (a) $a = -a_C$, $v_x = -a_C t$, $x = -(1/2)a_C t^2$

1.2.3 Questions and Problems: Einstein's Postulates and Time Dilation

17. (a) 2.5 s
(b) $\Delta t = 1.5 \text{ s}$
19. 7.57 years
21. 0.99995 c
23. 8.7 s
25. $7 \times 10^{-6} \text{ s}$
29. (a) $5.56 \times 10^{-10} \%$
(b) $3.12 \times 10^{-6} \%$

- (c) 1.3×10^7 m/s
- (d) A factor of 7100
- 31. (a) $\sqrt{1 + (L/h)^2}$
- (b) $h = c\Delta t$, $L = u\Delta t'$
- (c) $\Delta t'/\Delta t = \sqrt{1 + [u\Delta t'/(c\Delta t)]^2}$

1.3.2 Questions and Problems: Length Contraction and Simultaneity

- 9. (a) after
(b) before
- 13. (a) 99.99999687 yards
(b) 99.5 yards
(c) 44 yards
- 15. (a) L
(b) 0
- 19. (a) $u = (2\sqrt{2}/3)c$
(b) $(2\sqrt{2}/3)$ light-seconds
(c) $(2\sqrt{2}/9)$ light-seconds
(d) $(1/3)$ s
(e) $(1/9)$ s
(f) $(8/9)$ s
- 21. (a) 0.0048 c
- 23. (a) $L_{app} = L / [\gamma(1 + u/c)]$
- 25. (a) 20 years
(b) 9 light-years
(c) 8.72 years
(d) 4.36 years, 10 years
(f) 1.9 years
(g) 18.1 years
(h) 4.36 years
(i) 8.72 years
(j) 1.9 years
(k) 20 years

1.4.2 Questions and Problems: The Lorentz Transformations

- 9. 7.56 min, 18.9 light-min
- 11. (a) 33.5 years
- 21. (a) $x' = \gamma(x - ut)$
(b) $x = \gamma(x' + ut')$

1.5.3 Questions and Problems: Velocity Transformations and the Doppler Effect

- 7. (a) $0.5 c$
(b) $0.78 c$
(c) 1849 THz
- 9. (a) $0.38 c$
(c) $0.38 c$
- 11. 1.421826 GHz
- 15. (a) $(u_1 + u_2)/(1 + u_1 u_2/c^2)$
(b) $(4/5)c$
- 19. (a) 20 years
(b) 9 light-years
(c) 8.7 years
(d) 3.923 light-years
(e) 4.359 years
(f) 1.900 years
(g) $0.9945 c$
(h) $0.0945 c$
(i) 41.51 years
(j) 18.10 years and 4.348 years
(k) 20 years and 8.7 years

2

Relativity II: Dynamics

2.1.3 Questions and Problems: Spacetime Diagrams

- 15. (a) A-B is timelike, A-C is spacelike, and B-C is lightlike.
(b) A-B: $\sqrt{8}$ ls, A-C: $i\sqrt{8}$ ls, and B-C: 0
(c) $\sqrt{8}$ s
- 19. (b) 5 s
(c) Less
(d) Less
- 23. (a) 4 years
(d) 5, 3.2

2.2.3 Questions and Problems: Momentum and Energy

- 11. (a) 90 billion Joules
(b) 900 gallons
- 13. (a) 1.78×10^{-16} kg
(b) $\gamma = 10^{11}$
(c) 29 seconds
- 15. $v_A = -(1/26)c, v_B = (11/14)c$
- 17. (a) $\gamma \approx 1 + \frac{1}{2} \frac{v^2}{c^2} + \frac{3}{8} \frac{v^4}{c^4}$
(b) $\vec{p} \approx m\vec{v}$
(c) $\vec{p} \approx m\vec{v} + \left(\frac{mv^2}{2c^2}\right)\vec{v}$
(d) $E \approx mc^2 + (1/2)mv^2$
(e) $E \approx mc^2 + (1/2)mv^2 + (3/8)m(v^4/c^2)$

2.3.3 Questions and Problems: Mass and Energy (and the Speed of Light Squared)

15. (a) $-(15/17)c$
(b) $(25/8)mc^2$
(c) $-(3/5)c$
(d) $(25/8)mc^2$
(f) $(5/2)m$
17. (a) $(8/3)mc^2$
(b) $(4/3)mc$
(c) $M = (4/\sqrt{3})m, V = c/2$
19. $7 \times 10^{13} \text{ J}$
21. No change in mass. $\Delta p \approx 0.756 mc, \Delta E \approx 0.512 mc^2$
23. (a) $v = 10 \text{ m/s}$
(b) $150,000 \text{ kg m}^2/\text{s}^2$
(c) $1,350,000 \text{ kg m}^2/\text{s}^2$
(d) $\Delta m = 1.33 \times 10^{-11} \text{ kg}$
25. $E_{\text{ph}}c/(E_{\text{ph}} + m_0c^2)$
27. $30,000 \text{ m/s}$
29. $\theta \approx 36.87^\circ$

2.4.3 Questions and Problems: Four-Vectors

15. $\mathbf{P} = (2m_e c / \sqrt{3}, m_e c / \sqrt{3}, 0, 0)$
17. (a) $-(35/37)c$
(b) $E'_B = (37/12)mc^2, p'_B = -(35/12)mc$
(c) $((5/3)mc, -(4/3)mc, 0, 0)$
(d) $((37/12)mc, -(35/12)mc, 0, 0)$
19. mc
21. (a) $E' = \gamma(E - up_x), p'_x = \gamma(p_x - uE/c^2)$
25. (a) $(\gamma mc, \gamma mv)$
27. (b) $\mathbf{F} = \gamma \vec{\mathbf{F}}$

2.5.2 Questions and Problems: The Michelson–Morley Experiment

9. (a) $2.7 \times 10^{-9} \text{ m}$
(b) 10^{-8} periods

3

The Quantum Revolution I: From Light Waves to Photons

3.1.2 Questions and Problems: Interference

- 15. (b) Constructive
(d) Destructive
(f) Constructive
- 17. (b) 2π (or any multiple of it)
(c) π (or any odd multiple of it)
- 21. (a) One possible answer is:

$$y(x) = \begin{cases} 0 & 0 \leq x \leq 2\pi \\ -\sin x & 2\pi < x < 3\pi \\ 0 & 3\pi \leq x \leq 6\pi \end{cases}$$

- (b) Destructive
- 23. Constructive
- 25. ≈ 0.113 m
- 27. (a) $\lambda = \sqrt{26} - 5 \approx 0.1$ m
(b) $\lambda = 2(\sqrt{26} - 5) \approx 0.2$ m
- 29. (a) $A = 1/3$, $\lambda = 6\pi$, $k = 1/3$

3.2.3 Questions and Problems: The Young Double-Slit Experiment

- 9. (a) Possible answers include 0, 3 cm, and 6 cm.
(b) Possible answers include 1.5 cm, 4.5 cm, and 7.5 cm.
- 11. Destructive
- 15. (a) $\lambda L / (4d)$
(c) 2.8 mm
- 17. (a) destructively
(b) destructively
(c) dark

3.4.3 Questions and Problems: Blackbody Radiation and the Ultraviolet Catastrophe

11. 1.7×10^{19} photons per second
13. (a) 2/3, or 67%
(b) 4/5, or 80%
15. $4\pi a/(c^3 b^3)$
17. (a) $4.6k_B T$
(b) $0.55k_B T$
19. (a) $E_w = 0.15711k_B T$
(b) $E_w = 0.15719k_B T$
21. $E_w = 0.90k_B T$
23. $k_B T$
27. (a) $1.2 \times 10^{10} \text{ J/m}^3$
(b) $\approx 1/4,000,000$
(c) 46%
31. $1.7 \times 10^{13} \text{ Hz}$, infrared
33. (a) $T \approx 2900 \text{ K}, 2600^\circ\text{C}, 4800^\circ\text{F}$.
35. $8\pi v^2 k_B T / c^3$
37. $\rho = 8\pi hc \int_0^\infty \frac{1}{\lambda^5 (e^{hc/(k_B \lambda T)} - 1)} d\lambda$
(a) $p = a\pi/L$
(b) $k = b\pi/L, q = d\pi/L$
(c) $v = \frac{c}{2L} \sqrt{a^2 + b^2 + d^2}$
(d) $E_{\text{total}} = 2 \int_0^{\pi/2} \int_0^{\pi/2} \int_0^\infty E_w(r, \theta, \phi) r^2 \sin \theta dr d\theta d\phi$
(e) $\pi \int_0^\infty r^2 E_w(r) dr$
(f) $\frac{8L^3\pi}{c^3} \int_0^\infty v^2 E_w(v) dv$

3.5.3 Questions and Problems: The Photoelectric Effect

11. (a) $K_{\max} = h\nu - w_0$
13. (a) $1.6 \times 10^{-19} \text{ J}$
(b) $8.0 \times 10^{14} \text{ Hz}$
15. (a) 0.25 s
(b) No
17. (a) $4 \times 10^{-19} \text{ J}$

- (b) 1.5×10^{20}
 (c) 7.5×10^{21}
19. (a) 10^{15} Hz
 (c) 4.25 eV
21. Between 6.598×10^{-34} and 6.610×10^{-34} J. This doesn't quite include the modern value but is very close.

3.6.2 Questions and Problems: Further Photon Phenomena

13. 47,000 Kelvin
19. 0%, 1%, and 3.42%, respectively.
21. 5.877×10^{-11} m, 6.33×10^{-11} m
23. 4.1×10^{-26} J
25. 3 nm
27. 2×10^{-15} J
29. $E_C \approx E_{ph}/3400$
31. 298,772,069 m/s
33. (a) $E = mc^2/\sqrt{1 - v^2/c^2}$
 (b) approaches ∞
35. $v_f = \frac{m_e c^2}{m_e c^2 + h\nu_0(1 - \cos\theta)} v_0$
37. 2.2×10^{20} Hz

4

The Quantum Revolution II: Matter and Wavefunctions

4.1.2 Questions and Problems: The Bohr Model

15. $R_H/4 \approx 2.75 \times 10^6 \text{ m}^{-1}$
17. 118,000 K
19. 0.03%
23. (a) $v = e/\sqrt{4\pi\epsilon_0 m_e r}$
(b) $L = e\sqrt{m_e r/(4\pi\epsilon_0)}$
(c) $r = 4\pi\epsilon_0 n^2 \hbar^2 / (e^2 m_e)$, $v = e^2 / (4\pi\epsilon_0 n \hbar)$, $KE = m_e e^4 / (32\pi^2 \epsilon_0^2 n^2 \hbar^2)$
25. (a) $\Delta E = \frac{m_e e^4}{2(4\pi\epsilon_0)^2 \hbar^2} \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$
(b) $f = \frac{m_e e^4}{64\pi^3 \epsilon_0^2 \hbar^3 c} \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$
(c) $R_H = \frac{m_e e^4}{64\pi^3 \epsilon_0^2 \hbar^3 c}$
27. (b) $E = (1/2)n\hbar\omega/\pi$

4.2.2 Questions and Problems: Matter Waves

13. (a) 7 Million m/s
(b) $\approx 140 \text{ V}$
15. $\lambda_{\text{electron}} = 6.23 \times 10^{-9} \text{ m}$, $\lambda_{\text{neutron}} = 1.45 \times 10^{-10} \text{ m}$
17. (a) $1.2 \times 10^{-5} \text{ m}$
(b) $1.2 \times 10^{-7} \text{ m}$
19. $8.7 \times 10^{-10} \text{ m}$
21. $6.35 \times 10^{-34} \text{ kg m}^2/\text{s} - 6.99 \times 10^{-34} \text{ kg m}^2/\text{s}$. Yes.
23. $1.2 \times 10^8 \text{ V}$
25. (a) $\sqrt{h^2 + (x + w/2)^2} - \sqrt{h^2 + (x - w/2)^2}$
(b) $L = xw/\sqrt{h^2 + x^2}$
(c) $L = w \sin \phi$

4.3.3 Questions and Problems: Wavefunctions and Position Probabilities

11. $1/\sqrt{3}$
 13. (a) $\pm\sqrt{23}/6$
 (b) $i\sqrt{23}/6$
 (c) $23/36$
 15. (a) Not normalizable
 (b) $\sqrt{2/k}$
 17. $1/\sqrt{L}$
 19. $\sqrt{k/\pi}$
 21. (a) 2
 (b) $3/4$
 (c) $1/25$
 (d) 2
 23. (a) $1,000,000P_1$

4.4.2 Questions and Problems: The Heisenberg Uncertainty Principle

13. (a) x_0
 (b) 0
 (d) One over distance squared.
 (f) ψ_1, ψ_2 .
 15. (a) $\Delta p = \hbar/L$
 (b) $\hbar/(2L)$
 (c) $\hbar^2/(8mL^2)$
 17. (b) $L/\sqrt{3}$
 19. (b) $-e^2/(4\pi\epsilon_0 R)$
 (c) $\Delta p_x \geq \hbar/(2R)$
 (d) $\langle p_x \rangle = \hbar/(4R)$
 (e) $KE = 3\hbar^2/(32m_e R^2)$
 21. (a) $\Delta E \geq 2 \times 10^{-6} \text{ eV}$
 (b) $\Delta \nu \approx 5 \times 10^8 \text{ Hz}$
 23. (a) $\Delta x = \frac{1}{2\sqrt{k}}$
 (b) $\hbar\sqrt{k}$
 (c) $\hbar/2$

5

The Schrödinger Equation

5.1.2 Questions and Problems: Force and Potential Energy

9. (d) $-GM_{\text{Sun}}m_{\text{object}}/r^2$

11. (a) $d^2x/dt^2 = -k/m$

(b) $x = -k/(2m)t^2 + At + B$

(c) $x = -k/(2m)t^2 + x_0$

13. (a) J/m^2 , or kg/s^2

(b) $F = -kx$

(d) $x = 0$

5.2.2 Questions and Problems: Energy Eigenstates and the Time-Independent Schrödinger Equation

11. (a) E_4

(b) (i) $3/10$

(ii) 0

(c) $\int_{-\infty}^{\infty} A_5^2 |\psi_5(x)|^2 dx = 1$

(d) $|c_1|^2 + |c_2|^2 + |c_3|^2 + |c_4|^2 + |c_5|^2 = 1$

13. (a) $A = 1/\sqrt{5}$, $P(E = E_1) = 1/5$.

(b) $B = 1/\sqrt{3}$, $P(E = E_1) = 1/3$.

15. (a) $U = (1/2)kx^2$

(b) $-\frac{\hbar^2}{2m} \frac{d^2\psi}{dx^2} + \frac{1}{2}kx^2\psi(x) = E\psi(x)$

17. (a) $E = (3\hbar/2)\sqrt{k/m}$

(b) $\int_{-\infty}^{\infty} B^2 x^2 e^{-\sqrt{km} x^2/\hbar} dx = 1$

(c) $E = (\hbar/2)\sqrt{k/m}$ with probability $2/7$ and $E = (3\hbar/2)\sqrt{k/m}$ with probability $5/7$

19. (a) $E = U_0$

(b) No

(c) No

5.3.3 Questions and Problems: The Infinite Square Well

11. $\psi(x) = \sqrt{1/L} \sin(n\pi x/(2L))$, $E = \hbar^2\pi^2n^2/(8mL^2)$
13. (a) $(1/2)\psi_1 + (1/\sqrt{2})\psi_2 + (1/2)\psi_3$
(b) $E = \pi^2\hbar^2/(2mL^2)$ with probability $1/4$, $E = 2\pi^2\hbar^2/(mL^2)$ with probability $1/2$, and $E = 9\pi^2\hbar^2/(2mL^2)$ with probability $1/4$
(c) The same energy you found the first time.
17. $A = \sqrt{2/L}$

5.4.3 Questions and Problems: Other Bound States

11. (a) $\frac{d^2\psi}{dx^2} + (2E - x^2)\psi = 0$
17. (a) zero
(b) $0 < x < L : \psi(x) = A \sin\left(\frac{\sqrt{2mE}}{\hbar}x\right) + B \cos\left(\frac{\sqrt{2mE}}{\hbar}x\right)$
 $x \geq L : \psi(x) = Ce^{x\sqrt{2m(U_0-E)/\hbar}} + De^{-x\sqrt{2m(U_0-E)/\hbar}}$
(e) $\tan\left(\frac{\sqrt{2mE}}{\hbar}L\right) = -\sqrt{\frac{E}{U_0 - E}}$
19. (b) 1.98×10^{-7} eV

5.5.3 Questions and Problems: Complex Numbers

11. $2\sqrt{2}i$
13. (a) $-1 + 2i$
(b) $\sqrt{5}$
(c) $(4 - 7i)/(2 + 3i)$
(d) 5
(e) $\sqrt{5}$
15. (a) $1 + 7i$
(b) $|p| = |z_1||z_2|$
(c) $\phi_p = \phi_{z_1} + \phi_{z_2}$

5.6.3 Questions and Problems: Time Evolution of a Wavefunction

11. (a) $\sqrt{2/L} \sin(2\pi x/L)e^{-2i\hbar\pi^2 t/(mL^2)}$
(b) $(2/L) \sin^2(2\pi x/L)$
(c) $2\hbar^2\pi^2/(mL^2)$
(a) $\sqrt{2/L} \left[(1/2) \sin(2\pi x/L)e^{-2i\hbar\pi^2 t/(mL^2)} + (\sqrt{3}/2) \sin(5\pi x/L)e^{-25i\hbar\pi^2 t/(2mL^2)} \right]$

5 The Schrödinger Equation

- (b) $\frac{1}{L} \left[\frac{1}{2} \sin^2 \left(\frac{2\pi}{L} x \right) + \frac{3}{2} \sin^2 \left(\frac{5\pi}{L} x \right) + \sqrt{3} \sin \left(\frac{2\pi}{L} x \right) \sin \left(\frac{5\pi}{L} x \right) \cos \left(\frac{21\hbar\pi^2}{2mL^2} t \right) \right]$
- (c) $4mL^2/(21\hbar\pi)$
- (d) $2\hbar^2\pi^2/(mL^2)$ with probability 1/4 and $25\hbar^2\pi^2/(2mL^2)$ with probability 3/4
15. $|\Psi(x, t)|^2 = A^2 \left(\psi_1^2 + \psi_2^2 - 2 \cos \left[\frac{t(E_1 - E_2)}{\hbar} \right] \right)$

6

Unbound States

6.1.3 Questions and Problems: Standing Waves, Traveling Waves, and Partial Derivatives

15. (a) $5 \sin(2\pi t)$
(f) $\lambda = 6$, $T = 1$, $k = \pi/3$, $v = 1$
17. (a) 4
(b) 0
(c) 2
(d) -16
19. (a) $\lambda = 0.5$, $k = 4\pi$, $v = 1/10$
(b) negative
(c) zero
(d) $y(x) = 15 \sin(4\pi x) \cos(\pi t/5)$
21. 0.78 m
25. (a) $y_R(x, t) = A \sin(kx - \omega t)$
(b) $y_L(x, t) = A \sin(kx + \omega t)$
(d) $x = n\pi/k$
27. (d) $k = n\pi/L$

6.2.3 Questions and Problems: Free Particles and Fourier Transforms

15. (a) $A = 1/\sqrt{2L}$
(b) $\hat{\psi}(k) = -\frac{1}{2ki\sqrt{\pi L}} (e^{-ikL} - e^{ikL})$
(c) $\hat{\psi}(k) = \sin(kL)/\sqrt{\pi L k^2}$
(f) $\int_{-2/L}^{2/L} \frac{\sin^2(kL)}{\pi L k^2} dk$
(g) 86%

6.3.2 Questions and Problems: Momentum Eigenstates

11. (a) $\psi_1(x) = -(i/\sqrt{2L})(e^{ip_1x/\hbar} - e^{ip_2x/\hbar})$ where $p_1 = \pi\hbar/L$ and $p_2 = -\pi\hbar/L$
 (b) $p = \pm\pi\hbar/L$ with equal probability
 (c) $\pi\hbar/L$
13. (a) One example is $e^{-(3x/L)^2}$.
 (b) Closer
 (c) $[18/(\pi L^2)]^{1/4}$
 (d) Higher
 (e) $[L^2/(18\pi)]^{1/4} e^{-L^2k^2/36}$
 (g) 25%

6.4.3 Questions and Problems: Phase Velocity and Group Velocity

11. (a) $v_g = (1/2)\sqrt{g/k}$
 13. (a) both 1
 (b) $|\psi(x)|^2 = 2 + 2 \cos(0.1\pi x)$

6.5.3 Questions and Problems: Scattering and Tunneling

17. (a) $A + B = C, A - B = iC\sqrt{(U_0 - E)/E}$
19. (c) $\psi(x) = \begin{cases} Ae^{i(\sqrt{2m(E-U_0)/\hbar})x} + Be^{-i(\sqrt{2m(E-U_0)/\hbar})x} & x < 0 \\ Ce^{i(\sqrt{2mE/\hbar})x} + De^{-i(\sqrt{2mE/\hbar})x} & x \geq 0 \end{cases}$
 (d) D
 (e) $A + B = C, A - B = C\sqrt{E/(E - U_0)}$
 (a) $\left(\frac{\sqrt{1-U_0/E}-1}{\sqrt{1-U_0/E}+1}\right)^2$
 (d) 1
21. (e) $|B/A|^2$
 23. (b) $1 - \left|\frac{B}{A}\right|^2 = \frac{4E(E - U_0)}{4E(E - U_0) \cosh^2(\sqrt{2m(U_0 - E)L/\hbar}) + (2E - U_0)^2 \sinh^2(\sqrt{2m(U_0 - E)L/\hbar})}$

6.6.3 Questions and Problems: The Time-Dependent Schrödinger Equation

9. $f(x, t) = (A \sin[kx] + B \cos[kx]) (C \sin[(k/\sqrt{\beta})t] + D \cos[(k/\sqrt{\beta})t])$

7

The Hydrogen Atom

7.1.2 Questions and Problems: Quantum Numbers of the Hydrogen Atom

7. (a) $-1/16$ Ry, or about -0.850 eV, or -1.4×10^{-19} J
- (b) $\sqrt{12} \hbar$, or 3.65×10^{-34} kg · m²/s
- (c) $-3 \hbar$, or -3.16×10^{-34} kg · m²/s
- (d) $3/16$ Ry, or 2.55 eV, or 4.1×10^{-19} J

7.2.3 Questions and Problems: The Schrödinger Equation in Three Dimensions

11. (a) 0.0016
- (b) 0.0016
- (c) 0.0079
13. (a) equally likely
- (b) $y > 0$
- (c) equally likely
17. (a) $(1/2)\psi_{3,1,4} + (1/\sqrt{2})\psi_{2,1,2} + (1/2)\psi_{1,4,3}$
- (b) $E = 13\pi^2\hbar^2/(mL^2)$ and $E = 9\pi^2\hbar^2/(2mL^2)$, each with probability $1/2$.
- (c) The same energy you found the first time.
21. (a) $-\frac{\hbar^2}{2m} \left(\frac{\partial^2 \psi}{\partial x^2} T(t) + \frac{\partial^2 \psi}{\partial y^2} T(t) + \frac{\partial^2 \psi}{\partial z^2} T(t) \right) + U\psi T = i\hbar\psi(x, y, z) \frac{dT}{dt}$
- (b) $-\frac{\hbar^2}{2m} \left(\frac{\partial^2 \psi / \partial x^2}{\psi} + \frac{\partial^2 \psi / \partial y^2}{\psi} + \frac{\partial^2 \psi / \partial z^2}{\psi} \right) + U = i\hbar \frac{dT/dt}{T}$
27. $f(x, y, z) = e^{-(k+p)y} z^p (Ae^{\sqrt{k}x} + Be^{-\sqrt{k}x})$
- 29.

$$\begin{aligned}\psi_{abc}(x, y, z) &= A_{abc} \sin\left(\frac{a\pi}{L}x\right) \sin\left(\frac{b\pi}{W}y\right) \sin\left(\frac{c\pi}{H}z\right) \\ E_{abc} &= \frac{\pi^2\hbar^2}{2m} \left(\frac{a^2}{L^2} + \frac{b^2}{W^2} + \frac{c^2}{H^2} \right)\end{aligned}$$

7.3.3 Questions and Problems: Spherical Coordinates

7. (a) $(r, \theta, \phi) = (3\sqrt{2}, \pi/2, \phi/4)$
 (b) $(r, \theta, \phi) = (\sqrt{2}, \pi/4, 0)$
9. (b) $\sin \phi = y/\rho, \cos \phi = x/\rho, \tan \phi = y/x, x^2 + y^2 = \rho^2$
 (c) $\sin \theta = \rho/r, \cos \theta = z/r, \tan \theta = \rho/z, z^2 + \rho^2 = r^2$
11. (a) $20r^2$
 (b) $f(x, y, z) = (x^2 + y^2 + z^2)^2$
 (c) $20x^2 + 20y^2 + 20z^2$
 (d) They are, because $x^2 + y^2 + z^2 = r^2$.

7.4.3 Questions and Problems: Schrödinger's Equation and the Hydrogen Atom

13. (a) $r^6 e^{-r/(2a_0)} (12a_0 - r) / (2949120 a_0^9)$
 (b) One
19. (a) 1
 (c) $2 + 2 \cos \phi$
21. (a) 0.3
 (b) 1/2
23. (a) 1/2
 (b) 1/2
 (c) x-axis
25. (a) 1/8
 (b) 1/8
 (c) 0.097
 (d) 0.048

7.6.2 Questions and Problems: Spin and the Problem of Measurement

7. (a) $\cos^{2N} [\pi/(4N)]$
 (b) 1

7.7.3 Questions and Problems: Splitting of the Spectral Lines

11. (a) 2.6×10^{-9} eV
13. (a) $KE \approx p^2/(2m) - p^4/(8m^3c^2)$
 (b) 13.6 eV
 (c) 2×10^{-4} eV

8 Atoms

8.1.2 Questions and Problems: The Pauli Exclusion Principle

7. (a) $\frac{\pi^2 \hbar^2}{4mR^2} \sum_{x=1}^{n_{\max}} \sum_{y=1}^{n_{\max}} \sum_{z=1}^{n_{\max}} (x^2 + y^2 + z^2)$
- (b) $E = \frac{\pi^2 \hbar^2 n_{\max}^5}{4mR^2}$
- (c) $M = 2n_{\max}^3 m_n$
- (d) $E = \frac{\pi^2 \hbar^2}{R^2} \left(\frac{M^5}{2^{11} m_n^8} \right)^{1/3}$
- (f) $E = \frac{\pi^2 \hbar^2}{R^2} \left(\frac{M^5}{2^{11} m_n^8} \right)^{1/3} - \frac{3GM^2}{5R}$
- (g) $\frac{5\pi^2 \hbar^2}{3(2^{8/3})Gm_n^{8/3} M^{1/3}}$
- (h) 7,578 m
- (i) 2.5×10^{-34} m

8.2.3 Questions and Problems: Energy Levels and Atomic States

11. (a) (i) 2
(ii) 1.5
3.4 eV
13. 1s, 2s, 2p, 3s, 3p, 4s, 3d, 4p, 5s, 4f, 5d, 6p, 7s, 5f, 6d, 7p
15. sodium

8.4.2 Questions and Problems: X-Ray Spectroscopy and Moseley's Law

7. (a) 3 to 2
- (b) $(13.6 \text{ eV})(Z - 10)^2 / 2^2$
- (c) $(13.6 \text{ eV})(Z - 18)^2 / 3^2$
- (d) $(13.6 \text{ eV}) \left(\frac{(Z - 10)^2}{2^2} - \frac{(Z - 18)^2}{3^2} \right)$

- (e) 36%
 - (f) Slope $\sqrt{(3.4 \text{ eV})/\hbar}$, x-intercept Z = 10
9. (a) lower
(b) higher
11. (b) $\lambda = 32 \text{ pm}$

9

Molecules

9.1.4 Questions and Problems: Ionic and Covalent Bonds

13. (a) $r = 8.1 \times 10^{-10} \text{ m}$
(b) 0.56 eV

9.2.2 Questions and Problems: Bonding and Antibonding States

1. (a) -13.6 eV
(b) -13.6 eV
9. (a) $U(x, y, z) = -\frac{e^2}{4\pi\epsilon_0} \left(\frac{1}{\sqrt{x^2 + y^2 + z^2}} + \frac{1}{\sqrt{(x - L)^2 + y^2 + z^2}} \right)$
- (b) $\psi_1 = \frac{1}{\sqrt{a_0^3\pi}} e^{-\sqrt{x^2+y^2+z^2}/a_0}$, $\psi_2 = \frac{1}{\sqrt{a_0^3\pi}} e^{-\sqrt{(x-L)^2+y^2+z^2}/a_0}$
- (c) A = 0.56, B = 1.10
(d) $E_S \approx -28.7 \text{ eV}$, $E_D \approx -18.1 \text{ eV}$
(e) 13.6 eV
(f) 1.5 eV

10

Statistical Mechanics

10.1.3 Questions and Problems: Microstates and Macrostates

- 9. 3/8
- 11. (a) 1/6
(b) 3/10
(c) 2/5
- 13. (a) 2^{15}
(b) 455
(c) Roughly 1 in 72
- 15. (a) 0.66
(b) 0.96
(c) 0.999999998, or $1 - 2 \times 10^{-10}$
- 17. 0.063

10.2.3 Questions and Problems: Entropy and the Second Law of Thermodynamics

- 13. (a) 1140
(b) $k_B \ln 1140 = 9.71 \times 10^{-23} \text{ J/K}$
(c) 184,756
(d) $k_B \ln 184756 = 1.67 \times 10^{-22} \text{ J/K}$
- 15. (a) 1
(b) 2.5
(c) 3.5
(d) 3.6
(e) 10
(f) 20,000
- 17. (a) 1
(b) 100
(c) About 75 million
(d) 5×10^{20}

19. (a) 3
 (b) 9
 (c) $\Omega_A \Omega_B$
 (d) $1.516 \times 10^{-23}, 3.032 \times 10^{-23}$

10.3.3 Questions and Problems: Temperature

13. (a) $3.45 k_B, 6.45 k_B$
 (b) $197.91\epsilon/k_B, 100\epsilon/k_B$
 (c) 600ϵ
15. (a) 0
 (b) 0
17. 35,000 J/K
19. $C = -\frac{(dS/dE)^2}{d^2S/dE^2}$
21. 0, 0
23. (a) $S_A = 0, S_B = k_B \ln 3$
 (b) ϵ in A, and 3ϵ in B
 (c) 1/2
 (d) 454 K
25. (a) $W + Q_C = Q_H$
 (b) $Q_C/T_C = Q_H/T_H$
 (c) $T_C/(T_H - T_C)$
 (d) 0.081 J
27. (a) $Q_H = W + Q_C$
 (b) $Q_C/T_C = Q_H/T_H$
 (c) $1 - T_C/T_H$
 (d) 0.1 J

10.4.3 Questions and Problems: The Boltzmann Distribution

11. 32%
13. 88%
15. (a) 5.7%
 (b) 429 K
17. (a) 9/13
 (b) 0.22, or 22%
19. (b) 28%
21. (a) $P(E_1) = \frac{e^{-E_1/(k_B T)}}{e^{-E_0/(k_B T)} + e^{-E_1/(k_B T)}}$

10.5.3 Questions and Problems: Some Applications of the Boltzmann Distribution

15. (b) 170,000 m/s
 17. 1.3 Million K
 19. (a) $0.015k_B T$
 (b) $0.94k_B T$
 (c) $k_B T$
 23. (a) $\sqrt{8k_B T/(\pi m)}$
 (b) $\sqrt{3k_B T/m}$
 25. $3.6 \times 10^{10} \text{ Hz}$

10.6.3 Questions and Problems: Quantum Statistics

11. (a) 0.60
 (b) 1/2
 (c) 0.40
 13. (a) 1/3
 (b) 1/2
 (c) 0
 15. (a) 2/9
 (b) 1/10
 (c) 1
 17. 2/3
 19. C
 21. 125,000
 23. (a) Occupied: $e^{-(E-\mu)/(k_B T)} / \mathcal{Z}$. Unoccupied: $1/\mathcal{Z}$
 (b)

$$\frac{1}{\mathcal{Z}} e^{-(E-\mu)/(k_B T)} + \frac{1}{\mathcal{Z}} = 1$$

$$e^{-(E-\mu)/(k_B T)} + 1 = \mathcal{Z}$$

- (b) $1 + e^{-(E-\mu)/(k_B T)}$
 25. (a) 1
 (b) 16
 (c) 28
 (d) 2×10^{-19}
 27. (a) 1/2
 (c) $\mu \rightarrow -\infty$
 29. (a) 0
 (c) μ approaches the ground state energy.
 31. (b) $N = \int_0^{\pi/2} \int_0^{\pi/2} \int_0^{\infty} \bar{n} n^2 \sin \theta \, dn \, d\theta \, d\phi$

- (c) $N = (\pi/2) \int_0^\infty \bar{n} n^2 dn$
 (d) $N = \frac{m^{3/2}V}{\pi^2 \hbar^3 \sqrt{2}} \int_0^\infty \bar{n} \sqrt{E} dE$
 (e) $g(E) = m^{3/2}V \sqrt{E} / (\pi^2 \hbar^3 \sqrt{2})$
 (f) Any type

10.7.2 Questions and Problems: Blackbody Radiation

9. 11,000 K
 11. 6×10^{-6} J
 15. (a) 822.7 sec
 (b) 29 hours
 17. 650 K
 19. (a) $P(N) = \frac{1}{Z} e^{-Nh\nu/(k_B T)}$
 (b) $Z = \sum_{N=0}^{\infty} e^{-Nh\nu/(k_B T)}$
 (c) $\langle E \rangle = \sum_{N=0}^{\infty} \frac{Nh\nu}{Z} e^{-Nh\nu/(k_B T)}$
 (d) $\langle E \rangle = h\nu / (e^{h\nu/(k_B T)} - 1)$
 21. (a) $c dt$
 (b) ρdV
 (b) $E = c\rho dA dt/4$
 (c) $\frac{2\pi h}{c^2} \int_0^\infty \frac{v^3}{e^{hv/(k_B T)} - 1} dv$

10.8.3 Questions and Problems: Bose–Einstein Condensation

9. (a) 0.67 N
 (d) $N = 0.59$

11

Solids

11.1.2 Questions and Problems: Crystals

- 9. (a) $-q^2/(2\pi\epsilon_0 R)$
 - (b) $q^2/(4\pi\epsilon_0 R)$
 - (c) $-q^2/(6\pi\epsilon_0 R), q^2/(8\pi\epsilon_0 R)$
 - (d) $\left(2 - \frac{2}{2} + \frac{2}{3} - \frac{2}{4} \dots\right) \frac{-q^2}{4\pi\epsilon_0 R}$
 - (e) $2 \ln 2$
- 11. (a) 2.13, 22%
 - (b) -0.866, 149%

11.2.2 Questions and Problems: Band Structure and Conduction

- 5. (a) 6.1×10^{-47}
 - (b) 1.2×10^{-23} .
- 7. (a) 0.67 m/s
 - (b) 1.39×10^6 m/s
 - (c) a little over 400 lattice spacings

11.3.3 Questions and Problems: Semiconductors and Diodes

- 7. (a) 5.7×10^{-10}
- (b) 5.7×10^{-10}
- (c) About 200,000

11.5.3 Questions and Problems: Why Do Crystals Have a Band Structure?

- 9. (a) less than
- (b) lower

11.6.2 Questions and Problems: Magnetic Materials

- 9. (a) 0.148 T
- (b) 5.94×10^{-24} J

11.7.3 Questions and Problems: Heat Capacity

9. (a) $24.9 \text{ J}/(\text{mol} \cdot \text{K})$
(b) $22.25 \text{ J}/(\text{mol} \cdot \text{K})$
(c) $22.18 \text{ J}/(\text{mol} \cdot \text{K})$
11. (a) $3Nk_B T$
(b) $3Nk_B$
(c) $3R$
15. 0

12

The Atomic Nucleus

12.1.3 Questions and Problems: What's in a Nucleus?

- 7. (a) 6 MeV
- (b) 5.5 MeV
- (c) Nickel (Iron would also be a reasonable answer.)
- 9. (c) About 40 MeV
- (d) Gamma ray
- 11. (a) $5.3 \times 10^{-20} \text{ kg m/s}$

12.2.2 Questions and Problems: Experimental Evidence for Nuclear Properties

- 3. (a) 12 MeV
- (b) 39 MeV
- 5. 38°

12.3.2 Questions and Problems: Nuclear Models

- 7. 19 MeV

12.4.3 Questions and Problems: Three Types of Nuclear Decay

- 15. $t_{1/2}(\ln 3)/(\ln 2)$
- 17. (a) $Q = K_\alpha (1 + m_\alpha/m_Y)$
- (b) $K_\alpha = Q(A_X - 4)/A_X$
- (c) 0.28 MeV
- 19. (b) $K_p = p_p^2/(2m_p)$, $K_e = p_e c$
- (c) $p^2/(2m_p) + pc = Q$
- (d) $p = m_p c \left[-1 + \sqrt{2(m_n - m_e)/m_p - 1} \right]$
- (e) $\frac{1}{2} \left(\sqrt{\frac{2}{m_p} (m_n - m_e) - 1} - 1 \right)$

12.5.3 Questions and Problems: Fission and Fusion

11. $n d\sigma_\alpha$
13. (a) 5.42×10^{26} MeV
(b) 8.5
(c) about 43,000 kg

13

Particle Physics

13.1.2 Questions and Problems: Forces and Particles

- 5. (a) $1.25 \times 10^{-14} \text{ J}$
- (b) $3.86 \times 10^6 \text{ m/s}$
- (d) About 4 billion years

13.2.3 Questions and Problems: The Standard Model

- 17. (a) $2 \sin^2 \theta \cos^2 \theta \left[1 - \cos \left(\frac{t\Delta(m^2)c^4}{2\hbar E_0} \right) \right]$
- (b) $2 \sin^2 \theta \cos^2 \theta \left[1 - \cos \left(\frac{L\Delta(m^2)c^3}{2\hbar E_0} \right) \right]$
- (c) $L = 6600 \text{ m}, P = 84\%$

13.3.2 Questions and Problems: Detecting Particles

- 7. $\approx 3 \times 10^{-24} \text{ s}$
 - 9. (a) 2.1 MeV
 - (b) 0.0031 cm
 - 11. (a) $1.2 \times 10^{-20} \text{ kg m/s}$
 - (d) $1.2 \times 10^{-18} \text{ kg m/s}$
 - (e) $1.2 \times 10^{-18} \text{ kg m/s}$
 - (f) $4.2 \times 10^{-10} \text{ J}$
 - (g) $1200 \text{ MeV}/c^2$
13. 1582 MeV

14

Cosmology

14.1.2 Questions and Problems: The History of the Universe

9. (a) $c^5/(\hbar G^2) = 5 \times 10^{96} \text{ kg/m}^3$
(b) $6.5 \times 10^{-25} \text{ m}$

14.2.3 Questions and Problems: How Do We Know All That?

11. (b) $2D \sin(\theta)/\sin(\Delta\theta)$
(c) 1400 light-years

14.3.3 Questions and Problems: Infinite Universe, Finite Universe, Observable Universe

9. (a) Δx
(b) $2\Delta x$
(c) Yes
11. (a) $dx/dt = Hx$
(b) $x = x_0 e^{Ht}$
13. (a) $\frac{A_E L}{4\pi R^2}$
(b) $A_E L \rho \Delta R$
(c) $\frac{A_E L}{16\pi R^2}$
(d) $A_E L \rho \Delta R$

14.4.3 Questions and Problems: The Friedmann Equations

9. (a) $k_{\max} = 8\pi G\rho a^2/(3c^4)$
11. (b) $r = \left(\frac{3\kappa c}{4} \sqrt{\frac{3M}{\pi}} t + C \right)^{2/3}$
(d) $C = 0$
(e) 25 billion years
13. $T \ll 4 \times 10^{12} \text{ K}$
15. $\rho \propto a^{-4}$

17. (a) $Hx dt$
(b) $Hx + c$
(c) $dx/dt = 2x/(3t) + c$
(d) $x(t) = 3ct + Ct^{2/3}$
(e) $3ct$
(f) $x(t) = 3ct$
(g) 41.4 billion light years

14.5.3 Questions and Problems: Dark Matter and Dark Energy

7. $P < -\rho/3$
9. (a) $P = -\rho$
11. $3c^2\Lambda/(8\pi G)$

