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1.1.1 Discovery Exercise: Galilean Relativity

Spaceman Spiff is hurtling through the solar system at 3 kajillion miles per hour. His enemy on a nearby planet shoots a deadly missile straight toward him at 4 kajillion miles per hour. Spiff's rocket can withstand an impact of 4.5 kajillion mph without harm; anything above that will puncture the hull.

1. Suppose the enemy is ahead of Spiff, so the missile and the rocket crash head-on. Does the missile penetrate the hull?

   See Check Yourself #1 at www.cambridge.org/felder-modernphysics/checkyourself
2. Suppose the enemy is behind Spiff, so the missile catches up and rear-ends the rocket. Does it penetrate the hull?
3. Suppose the missile is coming in from the side, and hits perpendicular to the rocket's direction of travel. Does it penetrate the hull?

Write your answers here:

1.2.1 Discovery Exercise: Einstein's Postulates and Time Dilation

The speed of light, generally represented by the letter c, is approximately 3×10^8 m/s.

Spaceman Spiff is floating motionless in space when a spaceship zooms past at speed $c/3$ (one third the speed of light). At the instant the ship passes him Spiff turns on his flashlight, pointing the same direction that the ship is traveling. The beam leaves the flashlight at c, the speed of light, in Spiff's reference frame.

1. After one second, the beam of light has traveled how far in front of Spiff?
2. After one second, the spaceship has traveled how far in front of Spiff?
3. So after one second, the beam of light has traveled how far in front of the spaceship?

   See Check Yourself #2 at www.cambridge.org/felder-modernphysics/checkyourself
4. Use your previous responses to answer the question: how fast is the light beam traveling in the reference frame of the spaceship?

Write your answers here:

1.5.1 Discovery Exercise: Velocity Transformations

A spaceship is traveling in the positive x direction at $(3/4)c$ relative to you.

1. On board the spaceship is the captain's chair, which in the frame of the spaceship is immobile. What is the speed of this chair relative to you?
2. The captain shines a flashlight forward (in the positive x direction). What is the speed of the light beam in the ship's reference frame? What is the speed of the light beam in your reference frame?

Now a crew member at the back of the ship launches a small missile toward the front of the ship. (Don't ask us why.) The speed of that missile, in the reference frame of the ship, is also $(3/4)c$.

Question: What is the speed of the missile in your reference frame?

3. What is the answer to that question according to Galilean relativity?
4. Why is that clearly not the right answer?
5. Without doing any calculations, you can see that the right answer is between what (lower bound) and what (upper bound)?

Write your answers here:

2.1.1 Discovery Exercise: Spacetime Diagrams

A "spacetime diagram" is a plot with time on the vertical axis and space on the horizontal axis. Notice that this is backward from how you are probably used to plotting $x(t)$! Figure 2.1 shows an example with two moving objects.

1. Do the two objects start at the same time, the same place, or both?
2. Which object is moving faster, the blue or the green? How can you tell?
3. Copy the sketch. (Don't worry about distinguishing the blue and green lines.) Sketch the path of a light beam on your diagram. Assume you are using relativistic units, measuring time in seconds and distance in light-seconds.

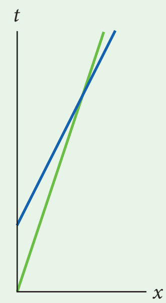

Figure 2.1 **A spacetime diagram for two moving objects.**

Write your answers here:

2.2.1 Discovery Exercise: Momentum and Energy

Consider the following scenario: an object starts at rest, and is subject to a constant force in the positive x direction.

1. In Newtonian mechanics, as you know, $\vec{F} = m\vec{a} = m(d\vec{v}/dt)$. Based on that equation, draw a quick sketch of this object's speed as a function of time. Then answer the question: what is $\lim_{t\to\infty} v$?
2. In relativity, as you know, speed can never reach or exceed c. Draw a second graph that copies your first graph for $v \ll c$, but obeys this universal speed limit in the long term.
3. If you keep accelerating an object, its kinetic energy will increase without bound (just like in classical physics). With that in mind, explain why the classical equation for kinetic energy $K = (1/2)mv^2$ cannot be correct in relativity.

As a bonus, you might want to think about how you could modify the function $K(v)$ so it would work in this limit. You don't have to know the correct answer, but think about whether you can find some function that would behave correctly.

Write your answers here:

2.3.1 Discovery Exercise: Mass

What does the word "mass" mean? If you ask introductory physics students that question – which we've done a lot, and it's a fun exercise – the most common answer is that mass is a measure of how many molecules (or atoms or fundamental particles) an object has. To push you beyond that (inadequate) answer, here's a more specific question.

Fact: A proton has roughly 2000 times the mass of an electron.

Question: What experiments could you do to test that fact, or to measure the mass ratio more precisely?

This is not a relativity-specific question, so feel free to give a purely classical answer. Also, don't feel constrained by practical considerations: assume you have an unlimited budget and unlimited technology. The goal is to articulate, in a measurable way, what that true fact means. (It does *not* mean that a proton is composed of 2000 bound electrons!) You can give a perfectly good answer to this question in one or two short sentences.

Write your answer here:

2.4.1 Discovery Exercise: Coordinate Transformation

Figure 2.16 is not a spacetime diagram. In fact, nothing in this exercise directly involves relativity.

You have just laid down some x and y axes to map out a space. Unfortunately your friend has laid down different axes, x' and y', rotated from yours by an angle θ (less than 90°).

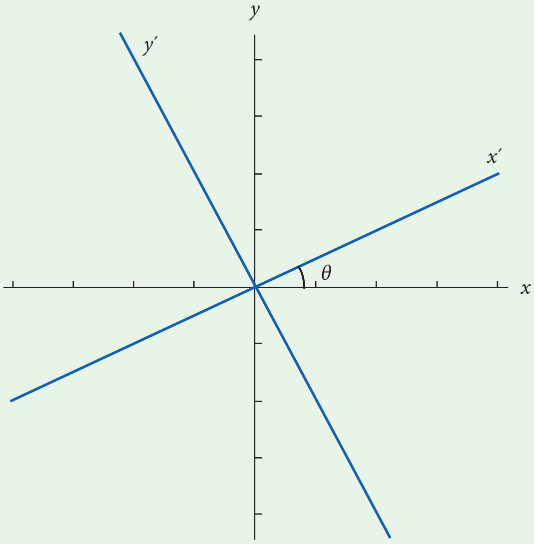

Figure 2.16 Two sets of axes rotated relative to each other.

You can convert any point from the unprimed coordinate system to the primed system by using the equations

$$\left.\begin{aligned} x' &= x\cos\theta + y\sin\theta \\ y' &= -x\sin\theta + y\cos\theta \end{aligned}\right\} \tag{2.5}$$

1. Look at the point $x = 1$, $y = 0$ on the drawing. Answer by looking at the diagram: Is its x' coordinate positive or negative? Is its y' coordinate positive or negative?
2. Confirm that Equations (2.5) match your visual prediction from Part 1.

Continued on next page

3. A bicycle rides smoothly across the page, and at a certain moment you measure everything about this bicycle in your unprimed coordinates. For each quantity specified below, indicate if you would use Equations (2.5) to *convert* your numbers to the primed coordinates, or if your numbers would be the *same* in the primed coordinates. For example, if we asked about the bicycle's position you would say that you would use Equations (2.5) to convert it, while if we asked about the height of the bicycle you would say that you and your friend agree about it; no conversion is needed.

(a) The bicycle's velocity

(b) The bicycle's speed

See Check Yourself #3 at www.cambridge.org/felder-modernphysics/checkyourself

(c) The bicycle's mass

(d) The bicycle's acceleration

Write your answers here:

2.5.1 Discovery Exercise: The Michelson–Morley Experiment

Ze and Maria are both capable of swimming with speed v in still water. A stream of width L is flowing with a steady current u (less than v) to the right.

Maria aims her body directly across the stream, swims to the other side, and then swims back. Note that her swimming velocity v is directed across the stream; she is also being carried downstream at speed u, but she doesn't care.

Ze, on the other hand, swims downstream a distance L, and then swims back upstream.

Both of their paths are shown in Figure 2.22. All speeds in this problem are far below the speed of light, so ignore relativistic effects.

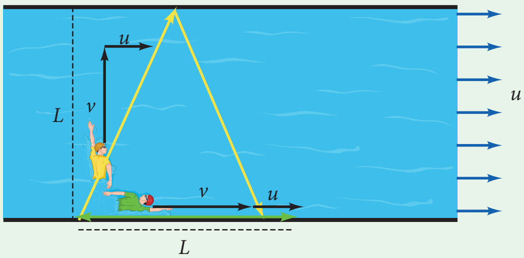

Figure 2.22 **Two swimmers moving along different paths.**

1. How long does Maria take to travel across the stream and back? *Hint*: The current is irrelevant for this question.
2. Ze's speed as he travels downstream is $v + u$. How long does it take him to travel distance L?
3. What is Ze's speed upstream, and how long does it take for him to return to his starting point?
4. Find the ratio of Ze's total time to Maria's. Simplify as much as possible.

This example introduces the basic idea of an interferometer. (It would be a more accurate analogy if Maria swam directly across instead of drifting downstream (Problem 8), but this conveys the basic idea with slightly simpler math.)

Write your answers here:

3.2.1 Discovery Exercise: The Young Double-Slit Experiment

Figure 3.9 looks straight down into a box filled with a shallow layer of water. The water at the bottom of the figure is being repeatedly struck by a small paddle, creating circular ripples that spread outward. These ripples pass through the narrow slits in Wall A. Whenever a wave passes through a narrow opening it spreads out circularly from there, so the slits in Wall A create two new circular ripples. The subject of this experiment is the impact of *those* ripples on Wall B.

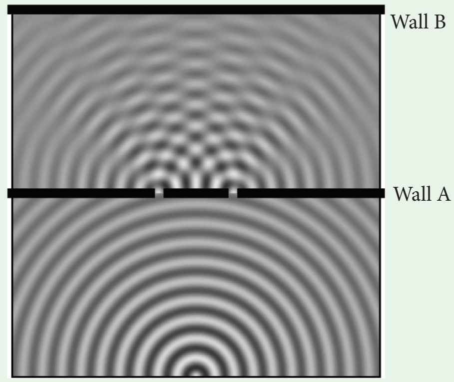

Figure 3.9 **A double-slit experiment with water.**

The two slits are equidistant from the paddle, which means the waves emanating from the two slits are perfectly in phase with each other.

Figure 3.10 shows the same box with two points marked on the back wall. It also shows the paths taken by the wave as it travels to those two points.

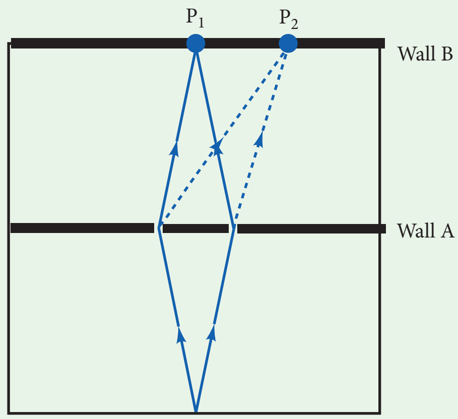

Figure 3.10 **Path lengths in the double-slit experiment.**

1. Point P_1 is in the middle of Wall B, and is therefore equidistant from the two slits. The solid lines in Figure 3.10 show the paths taken by the wave as it goes from the paddle, through each slit, and to Point P_1. Will the waves coming from the two slits reach P_1 "in phase" (leading to constructive interference) or "out of phase" (leading to destructive interference)?
2. The dashed lines in Figure 3.10 represent the paths to P_2 from each slit. Which of these two dashed paths is longer, the left or the right?
3. Suppose the longer path is longer by precisely half a wavelength. (That is certainly true at *some* point to the right of P_1.) Will the two waves reach this point in phase or out of phase?

   See Check Yourself #4 at www.cambridge.org/felder-modernphysics/checkyourself
4. Will there be another "in phase" spot to the right of P_2? Why or why not?

Before reading on, think about what it would look like if you graphed the amplitude of the wave as a function of position along Wall B. You can check your answer against the following Explanation.

Write your answers on the following page:

3.4.1 Discovery Exercise: Blackbody Radiation and the Ultraviolet Catastrophe

An enclosed cavity is filled with electromagnetic radiation, constantly being emitted and absorbed by the walls. The "spectrum" of that radiation tells how much of the energy is in the frequency range of blue light, how much in red, infrared, and so on.

Classical physics predicts that the spectrum in an enclosed cavity is $8\pi k_B T\nu^2/c^3$, where k_B and c are fundamental constants and T is the temperature. This function is called the "Rayleigh–Jeans spectrum." In 1900 Max Planck proposed a radical hypothesis – quantized energy levels – that led to a different formula, $\left(8\pi h\nu^3/c^3\right) / \left(e^{h\nu/(k_B T)} - 1\right)$. (Note the introduction of a new universal constant h.) These formulas can be written as:

$$S(\nu) = A\nu^2 \qquad \text{Rayleigh–Jeans (classical) spectrum} \qquad (3.2)$$

$$S(\nu) = \frac{B}{e^{C\nu} - 1}\nu^3 \qquad \text{Planck's spectrum} \qquad (3.3)$$

At 300 K (a typical room temperature), the constants are $A = 3.86 \times 10^{-45}$ J/(m^{3}Hz^3), $B = 6.17 \times 10^{-58}$ J/(m^{3}Hz^4), and $C = 1.6 \times 10^{-13}$ s.

1. Plot Equations (3.2) and (3.3) on the same graph, using the domain $0 \le \nu \le 2 \times 10^{12}$ s^{-1} and range $0 \le S \le 2 \times 10^{-20}$ J s/m^3. You should see that they track each other very well, but start to diverge as the frequencies get higher.
2. Plot Equations (3.2) and (3.3) on a second graph, using the domain $0 \le \nu \le 6 \times 10^{13}$ s^{-1} and range $0 \le S \le 3 \times 10^{-19}$ J s/m^3. For these higher frequencies you should see a dramatic difference.

The questions below are not asking for calculations; you can answer them quickly by looking at the graphs you just made.

3. Based on Planck's model, in roughly what frequency range would you expect to find the most radiation?
4. The energy density – the total energy in the cavity, divided by its volume – is obtained by integrating the spectrum function over all frequencies (0 to ∞). Explain why Planck's model might give a reasonable value for total energy, and the classical model cannot.

Write your answers on the following page:

3.5.1 Discovery Exercise: The Photoelectric Effect

This Discovery Exercise is part of a classical analysis of the photoelectric effect. As we shall see, the quantum mechanical understanding is quite different.

A laser beam with intensity 10 W/m^2 is shining on a plate coated with potassium. (Remember that one watt (W) means one joule per second.) An electron bound to a potassium atom requires an energy of roughly 2.3 eV (3.7×10^{-19} J) to be ejected from the atom.

1. Assume the atom has a circular cross-section with radius 10^{-10} m. How much power is striking the atom? Give your answer in watts.

   See Check Yourself #5 at www.cambridge.org/felder-modernphysics/checkyourself

2. Assume that all of the laser energy that strikes the atom is absorbed by the electron. How long will it take the electron to absorb enough energy to get ejected?

Write your answers here:

4.3.1 Discovery Exercise: Wavefunctions and Position Probabilities

A meter stick lies on the ground in front of you. You drop a pin that is guaranteed to land somewhere on that meter stick. Any point on the meter stick is exactly as likely as any other point.

1. Suppose the meter stick is marked with lines every centimeter: 1, 2, 3, . . . , 100. What is the probability that the closest line to your pin is the line marked 37? (This question is as trivial as it sounds.)
2. Now suppose the meter stick has lines every half-centimeter: 0.5, 1, 1.5, 2, . . . , 100. Now what is the probability that the closest line to your pin is the one marked 37?
3. Forget about the marks now. What are the odds that the pin lands *exactly* 37 cm from the end of the rod? (Assume the pin is infinitely small; it's a thought experiment.)

Write your answers here:

5.3.1 Discovery Exercise: The Simple Harmonic Oscillator Equation

The following differential equation is an example of a "simple harmonic oscillator" equation:

$$\frac{d^2y}{dx^2} = -y. \tag{5.3}$$

You can read Equation (5.3) as "The second derivative of the mystery function is the same as the original function, but multiplied by -1."

1. Show that the function $y = 0$ is a solution to Equation (5.3).
2. Show that the function $y = e^{-x}$ is *not* a solution to Equation (5.3).
3. Show that $y = \sin x$ is a solution to Equation (5.3).
4. Find another solution. (We're asking for any function that satisfies Equation (5.3) *other than* $y = 0$ or $y = \sin x$. But don't just write something down that looks good: test it and make sure it works!)
5. Finally, find a non-zero solution to the following differential equation:

$$\frac{d^2y}{dx^2} = -9y.$$

Equations such as Equation (5.3) can be used to model a *classical* simple harmonic oscillator. We will see in Section 5.4 that the equations for a *quantum* harmonic oscillator are more complicated. But in this section we will see how equations such as Equation (5.3) apply quantum mechanically to a simpler scenario, the "infinite square well."

Write your answers here:

5.4.1 Discovery Exercise: The Finite Square Well

A "finite square well" can be defined by a potential $U = 0$ for $0 < x < L$ and $U = U_0$ everywhere outside that region (Figure 5.6). A particle in this potential field is "bound" if $0 < E < U_0$.

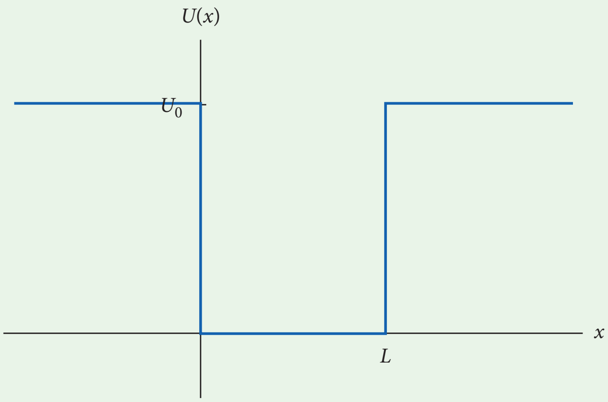

Figure 5.6 **A finite square well.**

In the region $x > L$, Schrödinger's equation becomes

$$\frac{d^2\psi}{dx^2} = \frac{2m(U_0 - E)}{\hbar^2}\psi. \tag{5.10}$$

We will express our solutions in terms of $\sqrt{U_0 - E}$ (a real quantity), not $\sqrt{E - U_0}$ (an imaginary quantity).

1. Two of the following functions are valid solutions of Equation (5.10) and the other two are not. Which two are? (Don't just guess: try them!)
   A. $\psi(x) = e^{\left(\sqrt{2m(U_0-E)}/\hbar\right)x}$
   B. $\psi(x) = e^{-\left(\sqrt{2m(U_0-E)}/\hbar\right)x}$
   C. $\psi(x) = \sin\left(\frac{\sqrt{2m(U_0 - E)}}{\hbar}x\right)$
   D. $\psi(x) = \cos\left(\frac{\sqrt{2m(U_0 - E)}}{\hbar}x\right)$
2. Of the two valid solutions you found, one of them is not a possible wavefunction for $x > L$. Which one, and why?

Write your answers here:

5.5.1 Discovery Exercise: The Complex Exponential Function

Equation (5.17) is the simplest example of the "simple harmonic oscillator (or SHO) equation,"

$$\frac{d^2y}{dx^2} = -y. \tag{5.17}$$

1. Show that $y = A\cos x + B\sin x$ is a solution to Equation (5.17).
2. Show that $y = Ce^{ix}$ is also a solution to Equation (5.17).

This differential equation, with its two very different-looking solutions, suggests a fundamental connection between complex exponential functions and real-valued sines and cosines.

Write your answers here:

5.6.1 Discovery Exercise: Time Evolution of a Wavefunction

In each question below we'll give you a complex number and ask you to calculate its modulus squared. Assume x is real, and simplify your answers as much as possible. You should be able to write each answer in terms of all real quantities (no i).

1. $z_1 = e^{ix}$, so $|z_1|^2 =$
2. $z_2 = e^{2ix}$ so $|z_2|^2 =$
3. $z_3 = z_1 + z_2 = e^{ix} + e^{2ix}$ so $|z_1 + z_2|^2 =$

   Hint: The answer is *not* the sum of the previous two answers, $|z_1|^2 + |z_2|^2$.

Write your answers here:

6.1.1 Discovery Exercise: A Traveling Wave

Consider the function $y(x, t) = 3\sin(2x + \pi t)$.

1. At the instant $t = 0$ this represents a wave $y(x)$ spread out along the x axis. What is the wavelength of that wave?
2. At the position $x = 0$ this represents an oscillation $y(t)$ going up and down over time. What is the period of that wave?

Write your answers here:

6.2.1 Discovery Exercise: Free Particles and Fourier Transforms

A particle with no forces on it can be described by the potential energy function $U(x) = 0$, for which the time-independent Schrödinger equation is $-(\hbar^2/(2m))\psi''(x) = E\psi(x)$.

1. Write the general solution to this Schrödinger equation using sines and cosines.

   See Check Yourself #6 at www.cambridge.org/felder-modernphysics/checkyourself
2. What problem can you see with these functions being the energy eigenstates for this system?
3. Rewrite the general solution to this Schrödinger equation using complex exponentials.
4. Does the complex exponential form have the same problem as the one you identified in Part 2? Why or why not?

Write your answers here:

6.4.1 Discovery Exercise: Speed of a Free-Particle Energy Eigenstate

Written in terms of momentum and energy, a free-particle energy eigenstate has the form

$$\Psi(x,t) = Ce^{i(px-Et)/\hbar}. \tag{6.11}$$

As we saw in Section 6.2, Equation (6.11) is a traveling wave.

1. Express this wave's velocity v_{wave} in terms of E and p. (If you need a reminder of the speed of a traveling wave, it's in Appendix F.)
2. Because a free particle has no potential energy, its energy is all kinetic: $E = (1/2)mv^2$. Using that equation and $p = mv$, re-express v_{wave} as a function of the particle's velocity v.

See Check Yourself #7 at www.cambridge.org/felder-modernphysics/checkyourself

You have just found a relationship between the velocity of a traveling wave and the velocity of a particle. But remember that the state of that particle (including its position) is entirely described by that wave! The fact that they are both moving, but with different velocities, is a vital hint to understanding how systems change in quantum mechanics.

Write your answers here:

6.5.1 Discovery Exercise: Scattering and Tunneling

Each question below gives a potential energy function. In each case a particle is coming in from the left ($x < 0$, $v > 0$) with energy E. Briefly describe how the particle would behave *classically*, assuming all forces are conservative. There is no quantum mechanics in this exercise.

1. $U(x) = 0$ for $x < 0$ and $U(x) = U_0$ for $x \geq 0$. Assume $E < U_0$.

   See Check Yourself #8 at www.cambridge.org/felder-modernphysics/checkyourself
2. The same potential function as Part 1, but this time $E > U_0$.
3. $U(x) = 0$ for $x < 0$ and $x > L$, and $U(x) = U_0$ for $0 \leq x \leq L$. Assume $E < U_0$.

In quantum mechanics the answer to the first question is more or less unchanged, but the other two scenarios come out very different from the classical expectation.

Write your answers here:

6.6.1 Discovery Exercise: A Partial Differential Equation

A "partial differential equation" is an equation that involves partial derivatives of a multivariate function; for example,

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial t}. \tag{6.18}$$

The function $f(x, t) = 2x + 2t$ is a solution to this equation because $\partial f/\partial x$ and $\partial f/\partial t$ both equal 2. The function $f(x, t) = xt$ is not a solution because $\partial f/\partial x = t$ and $\partial f/\partial t = x$, so they are not equal.

Which of the following are solutions to Equation (6.18)? (Choose all that apply.)

1. $f(x, t) = 5$
2. $f(x, t) = x - t$
3. $f(x, t) = x^2 + t^2$
4. $f(x, t) = x^2 + 2xt + t^2$

Write your answers here:

7.2.1 Discovery Exercise: The Two-Dimensional Infinite Square Well

The two-dimensional version of the infinite square well is a square box of side length L. A particle is free to move anywhere inside the box, but can never leave it (Figure 7.2).

The time-independent Schrödinger equation in 2D is just like the 1D version except it has spatial derivatives with respect to both x and y. Inside the box (where $U = 0$) it looks like

$$-\frac{\hbar^2}{2m}\left(\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}\right)=E\psi. \qquad (7.2)$$

The Explanation (Section 7.2.2) will show you how to solve such equations. Here we are going to skip to the solution:

$$\psi(x,y)=A\sin\left(\frac{a\pi}{L}x\right)\sin\left(\frac{b\pi}{L}y\right). \qquad (7.3)$$

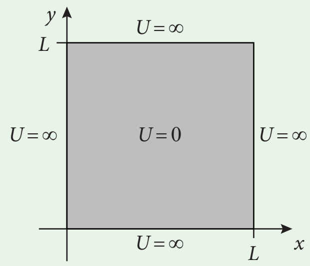

Figure 7.2 **A particle in a 2D box has potential energy $U = 0$ in the region $0 \le x \le L$, $0 \le y \le L$, and potential energy $U = \infty$ outside that region.**

Recall that at an infinite potential jump ψ may not be differentiable, but it must still be continuous. Because $\psi(x,y)$ must be zero outside the box, continuity gives us four boundary conditions, including "$\psi(x,y) = 0$ when $x = 0$" and "$\psi(x,y) = 0$ when $y = 0$." (We've already built those two into Equation (7.3); make sure you see that.)

1. Write the other two boundary conditions, similarly to how we just wrote the first two.
2. What must be true of the numbers a and b in order for Equation (7.3) to match the boundary conditions you wrote in Part 1?
3. Plug the solution (Equation (7.3)) into the differential equation (Equation (7.2)). Solve the resulting equation to find the energy (E) in terms of the quantum numbers a and b. *There should be no x or y in your answer.*

   See Check Yourself #9 at www.cambridge.org/felder-modernphysics/checkyourself
4. Based on your solutions to Parts 2 and 3, the lowest possible energy of this particle is $\pi^2\hbar^2/(mL^2)$. What are the next two energy levels?

Write your answers here:

7.3.1 Discovery Exercise: Polar Coordinates

If you want to specify the location of a point on a plane, you can use the "Cartesian coordinates" x and y. Alternatively, you can use "polar coordinates": ρ is the distance to the origin and ϕ is the angle measured from the positive x axis (Figure 7.6).[4]

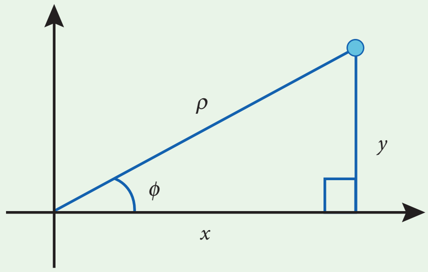

Figure 7.6 Coordinates of a point in Cartesian and polar coordinates.

1. Write functions for finding x and y if you are given ρ and ϕ. (You should be able to see these functions quickly from the diagram.)
2. Write functions for finding ρ and ϕ if you are given x and y. (Same comment.)

   See Check Yourself #10 at www.cambridge.org/felder-modernphysics/checkyourself
3. Draw the set of all points for which $2 \le \rho \le 3$.
4. Draw the set of all points for which $0 \le \phi \le \pi/2$.
5. The point $(5, \pi/2)$ is the same as the point $(5, 9\pi/2)$ in polar coordinates. Give one other (ρ, ϕ) combination that identifies this same point.

4 You may have learned polar coordinates with r and θ instead of ρ and ϕ. The letters aren't important; the meaning is the same.

Write your answers here:

7.4.1 Discovery Exercise: Schrödinger's Equation and the Hydrogen Atom

An electron orbiting a nucleus feels a potential energy $U = -k/r$.

1. Write the time-independent Schrödinger equation for the electron in spherical coordinates. (See Equation (7.15) on p. 335.)
2. Make a guess $\psi(r, \theta, \phi) = R(r)\Theta(\theta)\Phi(\phi)$, and separate variables to derive an ODE for $R(r)$.

Write your answers here:

7.7.1 Discovery Exercise: Splitting of the Spectral Lines

Here is the time-independent Schrödinger equation for a single electron orbiting a single proton:

$$-\frac{\hbar^2}{2m}\nabla^2\psi - \frac{e^2}{4\pi\epsilon_0 r}\psi = E\psi. \tag{7.24}$$

Unlike equations that represent more complicated systems (such as *two* electrons orbiting a nucleus), Equation (7.24) can be solved analytically. The math leads to the eigenstates and eigenvalues in Appendix G, and those formulas lead to predictions that hold up very well in the lab.

Very well . . . but not perfectly. Photons emitted when electrons drop down to other levels, and other experimental evidence, point to very small but consistent deviations from the energy levels $E_n = -(1/n^2)$ Ry.

Why? Can you think of an approximation we have made in this chapter, or a property of protons and electrons that we have not taken into account? Can you think of two or three?

Write your answer here:

8.2.1 Discovery Exercise: Energy Levels and Atomic States

A lithium atom has three protons and three electrons. The first two electrons are in the state $n = 1, l = 0$, while the third one has $n = 2, l = 0$.

1. In a hydrogen-like atom (only one electron), the energy of an eigenstate is $-(13.6\text{ eV})Z^2/n^2$, where Z is the number of protons in the nucleus. If the $n = 2$ electron were the only electron in the lithium atom, how much energy would it have?
2. In the actual lithium atom, the $n = 2$ electron feels forces from the other electrons as well as from the nucleus. Those $n = 1$ electrons act like a spherical cloud of charge at a smaller radius than the $n = 2$ electron (Figure 8.1). Taking into account the force from those inner electrons, would you expect the actual energy of the $n = 2$ electron to be higher (less negative), or lower (more negative), than your answer to Part 1? Why?

See Check Yourself #11 at www.cambridge.org/felder-modernphysics/checkyourself

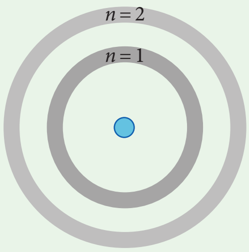

Figure 8.1 Lithium energy levels.

Write your answers here:

8.3.1 Discovery Exercise: The Periodic Table

Section 8.2 presented a simple model that is not numerically accurate but good for understanding qualitative trends. In this model the outermost electron in an atom feels the electric pull of the nucleus “screened” by other electrons. The electrons in its own subshell screen half a proton each, and the electrons in lower subshells screen a full proton each. The resulting “effective” charge goes into the formula for the energy required to liberate the outermost electron: $E = -(1\ \text{Ry})Z_{\text{eff}}^2/n^2$, where n is the principal quantum number of that electron.

In this exercise you will use this model to compare fluorine (ground state $1s^2 2s^2 2p^5$), neon (ground state $1s^2 2s^2 2p^6$), and sodium (ground state $1s^2 2s^2 2p^6 3s^1$).

1. Would you expect fluorine, neon, or sodium to be most likely to give up an electron?

You can also use this model to estimate how likely an atom is to accept an electron from another atom. If the extra electron would have a very low energy (very negative), the atom is likely to absorb that electron.

2. Would you expect fluorine, neon, or sodium to be most likely to accept an extra electron? Explain how you can answer this from our simple model.

Write your answers here:

9.1.1 Discovery Exercise: Ionic Bonds

When one atom gives an electron to another, they acquire opposite charges and attract each other. That attraction creates an "ionic bond."

As you answer the following questions about ionic bonds, keep your eye on Appendix H, and keep in mind what you learned in Chapter 8 about why some elements tend to give up electrons and others are inclined to absorb them.

1. Which of the following pairs of elements are likely to form an ionic bond? (Choose all that apply and briefly explain your answers.)
   - **A.** Li and Na
   - **B.** Li and F
   - **C.** F and Cl
2. Sometimes an atom gives one electron each to two other atoms and ionically bonds to both of them. Which of the following molecules could form this way? (Choose one.)
   - **A.** LiS_2
   - **B.** Li_2S
   - **C.** $BeCl_2$
   - **D.** Be_2Cl

Write your answers here:

9.1.2 Discovery Exercise: Covalent Bonds

Consider a simple model of a hydrogen atom as a proton surrounded by a thin spherical shell of negative charge at one Bohr radius. Nearby is a single bare proton. See Figure 9.1.

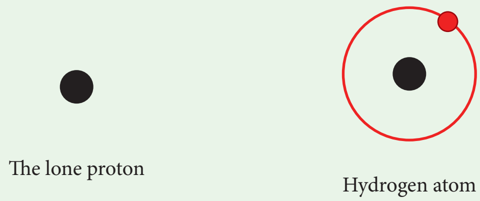

Figure 9.1 **A bare proton and a hydrogen atom.**

1. Does the bare proton feel a net force, and if so in which direction?
2. Does the hydrogen nucleus feel a net force, and if so in which direction?
3. Does the electron cloud feel a net force, and if so in which direction?
4. One moment later, sketch how the picture would have changed because of the forces you predicted.
5. In your new picture, does the bare proton feel a net force, and, if so, in which direction?

Write your answers here:

10.1.1 Discovery Exercise: Microstates and Macrostates

You roll two dice, one at a time, and write the results as an ordered pair. For example (1, 5) means you rolled a 1 and then a 5.

1. The results (1, 6) and (6, 1) each have a sum of 7. How many combinations (including those two and others) have a sum of 7?
2. How many combinations have a sum of 2?

   See Check Yourself #12 at www.cambridge.org/felder-modernphysics/checkyourself
3. After a while you have filled a sheet of paper with ordered pairs. Now you count how many pairs sum to 2, and how many pairs sum to 7. Which of these two sums do you expect to see more often? How much more often?

Write your answers here:

10.2.1 Discovery Exercise: Entropy and the Second Law of Thermodynamics

There are N air molecules in a room. Imagine that the molecules don't affect each other at all (a reasonable approximation), and that in each millisecond each molecule has a 10% chance of moving from the side of the room where it is at that moment to the other side. Initially all the air is on the left side of the room.

1. A millisecond later, about how many molecules are on the right side of the room?
2. A second later, about how many molecules are on the right side of the room?

   See Check Yourself #13 at www.cambridge.org/felder-modernphysics/checkyourself
3. How will you see the number of molecules on the right side change if you keep watching for several hours?

Write your answers here:

10.3.1 Discovery Exercise: Temperature

System S_1 has 5 objects, and System S_2 has 50 objects. Each object can have energy 0 or 1.

The initial macrostate of this system is "S_1 has a total energy of 1, and S_2 has a total energy of 3."

Then one unit of energy flows from S_2 to S_1.

1. Does the entropy of S_1 increase or decrease? By how much?
2. Does the entropy of S_2 increase or decrease? By how much?
3. Overall do you expect to see energy flowing from S_2 to S_1 as we described, which is from the higher-energy system to the lower? Or do you expect to see it flow the other way? *Hint*: Your answer to this part should be based on your answers to the other parts!

   See Check Yourself #14 at www.cambridge.org/felder-modernphysics/checkyourself

Write your answers here:

10.4.1 Discovery Exercise: The Boltzmann Distribution

A single paramagnetic atom (which we'll call A) is in contact with a system comprising 100 atoms. All 101 atoms can have energy 0 or ϵ, and the combined system has a total energy of 4ϵ.

1. How many accessible microstates does the entire combined system have?
2. In how many of those microstates does Atom A have energy 0, and in how many does it have energy ϵ?

   See Check Yourself #15 at www.cambridge.org/felder-modernphysics/checkyourself
3. Based on your answer, you can conclude that A is far more likely to have energy 0 than ϵ. Why does this not violate the fundamental assumption of statistical mechanics?

Write your answers here:

10.5.1 Discovery Exercise: A Perfectly Even State Density

Consider a system that has three possible energy levels: $E = 0$, $E = 3k_B T$, and $E = 6k_B T$. Each of those levels represents exactly one microstate, and their probabilities follow the Boltzmann distribution $P = (1/Z)e^{-E/(k_B T)}$.

1. Recall that the formula for expectation value is:

$$\langle E \rangle = \sum_E EP(E).$$

   Calculate the expectation value of energy for this system. Your answer should be in the form of a decimal times $k_B T$.

   See Check Yourself #16 at www.cambridge.org/felder-modernphysics/checkyourself

2. Explain briefly what your result means, in a sentence that starts "If you measured a million of these particles, . . ."

Write your answers here:

10.6.1 Discovery Exercise: Quantum Statistics

You flip three pennies.

1. List all of the possible outcomes. For example, HHT is one and HTH is another. Assuming all of these outcomes are equally likely, what is the probability of getting three heads?
2. List all the possible outcomes again, but this time only list how many heads and tails you got. For example, don't list HHT and HTH as separate states. Now, assuming all of *these* outcomes are equally likely, what is the probability of getting three heads?

   See Check Yourself #17 at www.cambridge.org/felder-modernphysics/checkyourself
3. Which of your two answers is the correct probability for getting three heads when you flip three pennies?

Write your answers here:

10.8.1 Discovery Exercise: Bose–Einstein Condensation

Note: You can answer the questions below with no calculations.

System S_D comprises 10 distinguishable particles, each of which can have energy 0, ϵ, 2ϵ, etc.

1. How many microstates of this system have a total energy of 0?
2. How many microstates of this system have a total energy of ϵ?

System S_B comprises 10 bosons, each of which can have energy 0, ϵ, 2ϵ, etc.

3. How many microstates of this system have a total energy of 0?
4. How many microstates of this system have a total energy of ϵ?

See Check Yourself #18 at www.cambridge.org/felder-modernphysics/checkyourself

5. Assuming both systems obey the Boltzmann distribution, which system would be more likely to have a total energy of zero? Briefly explain your answer.

Write your answers here:

11.3.1 Discovery Exercise: Semiconductors

Consider a wire made from a semiconductor. As we explained in Section 11.2, that word implies three things: there is an energy band that is completely full, the band immediately above it is completely empty, and the gap between these two bands is relatively small (Figure 11.11).

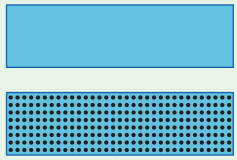

Figure 11.1 Bands in a semiconductor.

1. If a voltage difference is applied across this wire, very little current will flow. Briefly explain why.

Now imagine that the electricity fairy sprinkles a few extra electrons into the wire. (The fairy also increases the positive charge of the lattice so the wire remains electrically neutral.)

2. Where in Figure 11.11 will those electrons go? You can copy the figure and draw them in or explain in words where they will end up.
3. How does this change the resistance of the wire? Explain briefly why.

Write your answers here:

11.5.1 Discovery Exercise: Why Do Crystals Have a Band Structure?

The first few questions in this Discovery Exercise review bonding and antibonding states. If you have trouble with these questions, review that material in Section 9.2.

Figure 11.26 shows the four lowest energies for an electron in the vicinity of a single proton (aka a hydrogen atom). The electron has ground state energy −13.6 eV (the bottom line in the drawing). Because of spin, there are two states available at that energy.

Now consider an electron in the vicinity of two protons.

1. In the limit where those two protons are very far from each other, what is the ground state energy for the electron and how many states are available at that energy?
2. If the two protons are roughly 10^{-10} m apart (a typical molecular separation), how many states are available to the electron at roughly −13.6 eV? Is the energy of those states . . .
   - **A.** higher than −13.6 eV?
   - **B.** lower than −13.6 eV?
   - **C.** higher for some of the states and lower for others?
3. Sketch an energy level diagram similar to Figure 11.26 for an electron orbiting around two protons 10^{-10} m apart.

   See Check Yourself #19 at www.cambridge.org/felder-modernphysics/checkyourself

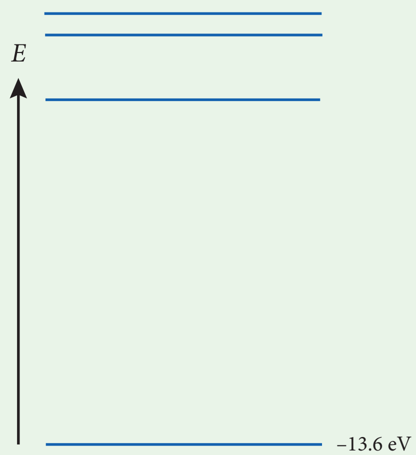

Figure 11.26 The first few energy levels of hydrogen.

These last questions extend beyond Chapter 9.

4. Now imagine an electron in the vicinity of three protons, all far from each other. How many ground states are available to that electron?

5. If you bring three hydrogen atoms very close together (don't worry for the moment about how you do this), what will the energy level diagram for an orbiting electron look like? It's OK if you get this one wrong, but make your best guess and write a sentence or two explaining your reasoning.

Write your answers here:

11.7.1 Discovery Exercise: Heat Capacity

A crystal contains N nuclei arranged in a lattice structure. Each nucleus has a fixed position in the lattice, but can make small vibrations around that position. Because those vibrations can occur in all three directions, these N nuclei can be considered as $3N$ simple harmonic oscillators.

1. According to the equipartition theorem (Appendix I), what is the total *thermal* energy of such a collection of oscillators?

   See Check Yourself #20 at www.cambridge.org/felder-modernphysics/checkyourself

2. Remember that "heat capacity" can be approximately defined as the amount of energy required to raise the temperature of an object by one degree. Based on your answer to Part 1, what is the heat capacity of this crystal?

Write your answers here:

12.1.1 Discovery Exercise: What's in a Nucleus?

Two protons sit 10^{-15} m away from each other. They are held together by the "strong nuclear force" but repel each other electrically.

1. Find the magnitude of the (positive) electric potential energy of the two protons.

   See Check Yourself #21 at www.cambridge.org/felder-modernphysics/checkyourself

2. For the protons to be bound in the nucleus, they must have a negative potential energy whose magnitude is larger than the electric potential energy you just calculated. To put that number in context, how many times larger is that electric potential energy than the 13.6 eV binding energy of an electron in a hydrogen atom?

Write your answers here:

12.4.1 Discovery Exercise: Three Types of Nuclear Decay

Table 12.1 lists binding energy per nucleon in MeV for several nuclides.

Consider the following nuclear reaction:

$$^{238}\text{U} \rightarrow {}^{234}\text{Th} + {}^{4}\text{He}.$$

1. Calculate the *total* binding energy (not binding energy per nucleon) before and after the reaction.

See Check Yourself #22 at www.cambridge.org/felder-modernphysics/checkyourself

2. Would you expect this reaction to occur spontaneously? Why or why not?

3. Answer the same questions for the reaction $^{84}\text{Kr} \rightarrow {}^{80}\text{Se} + {}^{4}\text{He}$.

Table 12.1 **Binding energy per nucleon for several nuclides**

Nuclide	^{238}U	^{84}Kr	^{234}Th	^{80}Se	^{4}He
B/A (MeV)	7.57	8.72	7.60	8.71	7.07

Write your answers here:

12.5.1 Discovery Exercise: Nuclear Fission

Consider the following fission reaction:

$$^{236}_{92}\text{U} \rightarrow \,^{92}_{36}\text{Kr} + \,^{141}_{56}\text{Ba} + \,?$$

1. What must be the final product (where we have left a question mark) to keep proton number and neutron number conserved?
2. Use Figure 12.4 to estimate the binding energy per nucleon of each of these three nuclides. Based on those numbers and your answer to Part 1, estimate the energy released by this reaction.

Write your answers here:

13.2.1 Discovery Exercise: The Standard Model

You have been given a construction kit with four kinds of pieces:

- The "down quark" has charge $-1/3$, and the "antidown quark" has charge $+1/3$.
- The "up quark" has charge $+2/3$, and the "antiup quark" has charge $-2/3$.

All the quarks in your set have spin 1/2, but with a flexible addition rule: two spin-1/2 particles can combine to a spin of either 0 or 1, depending on whether they are aligned or anti-aligned.

You have an unlimited supply of all four types. Show how you can combine quarks to make each of the following particles:

1. A proton (charge $+1$, spin 1/2)
2. A neutron (charge 0, spin 1/2)
3. A neutral pion (charge 0, spin 0)
4. A negatively charged pion (charge -1, spin 0)

Write your answers here:

13.4.1 Discovery Exercise: Symmetries

You are watching a video of a ball falling to the ground and then bouncing back up. But you suspect that the video has been tampered with. For each of the following tamperings, describe how you could detect it – or say that you could not.

1. The video has been rotated by 90°, so the "down" direction in real life is "right" in the video.

   See Check Yourself #23 at www.cambridge.org/felder-modernphysics/checkyourself
2. The video is being played backward.
3. The video has been speeded up.
4. The video has been left/right reflected.

Write your answers here:

14.2.1 Discovery Exercise: Distance to a Star

One night you measure a particular star that is exactly 90° away from the Sun in the sky. Six months later, when the Earth has moved to the opposite side of the Sun, you measure that the star is at 89.9999045° away from the Sun (Figure 14.1).

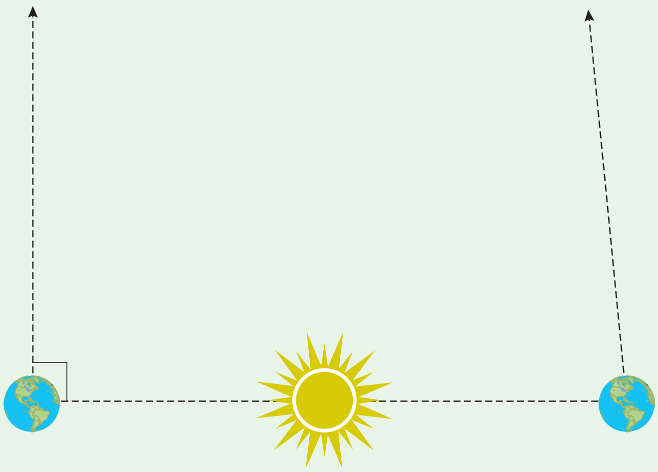

Figure 14.1 **From opposite sides of the Sun, the same star is seen at two different angles.**

Taking the distance from the Earth to the Sun as 100 million miles, calculate the distance to the star. Express your final answer in light-years. *Hint*: We didn't specify whether we were asking for distance from the Earth the first time, or the second time, or for distance from the Sun. Pick whichever one you want to calculate, and think about why we didn't have to specify which one we meant.

See Check Yourself #24 at www.cambridge.org/felder-modernphysics/checkyourself

Write your answer here:

14.3.1 Discovery Exercise: An Infinite, Expanding, One-Dimensional Universe

Boris and Natasha both live on a universe that looks suspiciously like an infinitely long ruler, marked off in inches. One day, for no obvious reason, the ruler stretches horizontally: *all horizontal distances on the ruler double, and this expansion takes exactly one second.* (See Figure 14.4.)

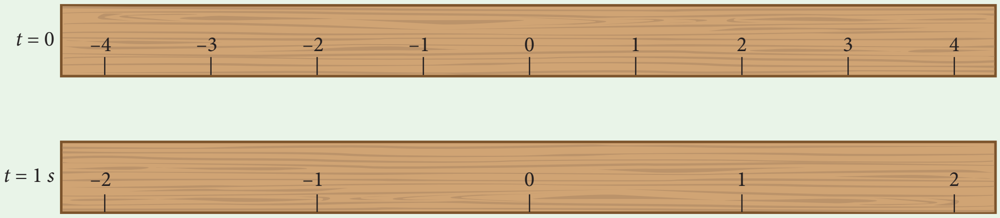

Figure 14.4 **Boris and Natasha's universe, before and after its great expansion.**

1. Boris is standing on Mark 0 when the great expansion happens. In one second he sees every other mark double its distance from him: for instance, Mark 2 used to be two inches to his right, but now it is four inches to his right. Write down the *average speed* that Boris calculates for Mark 2 during its one-second move.
2. Repeat Part 1 to find the speeds of Marks −2, −1, and 1.
3. The Hubble–Lemaître law in our universe states that the speed with which a galaxy is receding from us is directly proportional to its distance from us. Does Boris calculate the same law in his universe?

   See Check Yourself #25 at www.cambridge.org/felder-modernphysics/checkyourself
4. Natasha is standing on Mark 1. During that same time, what average speeds does she measure for Marks −1, 0, 2, and 3?
5. If Natasha considers herself the center of the universe (which believe us, she does), does she also observe the Hubble–Lemaître law – the speed with which a galaxy is receding *from her* is proportional to its distance *to her* – or does she require a modified law? Show how your answer follows from your calculations.

Write your answers here:

14.4.1 Discovery Exercise: The Friedmann Equations

Consider a uniform expanding sphere of matter with radius $r(t)$. Let Galaxy X be a galaxy on the outer edge of this sphere. In this exercise you are going to analyze the dynamics of this sphere using entirely Newtonian physics.

1. Write the net gravitational force on Galaxy X. Your answer will depend on the mass of the galaxy m_X and the mass of the entire sphere M_S.
2. Write the gravitational potential energy of Galaxy X.
3. Write an equation expressing conservation of energy for Galaxy X. Your equation should include r, $\dot{r}$ (which is shorthand for dr/dt), and an arbitrary constant.

   See Check Yourself #26 at www.cambridge.org/felder-modernphysics/checkyourself
4. Rewrite your equation so it depends on ρ, the mass per unit volume, rather than M_S.
5. Show that your equation can be rewritten in the following form with correctly chosen constants k_1 and k_2:

$$\left(\frac{\dot{r}}{r}\right)^2 = k_1\rho - \frac{k_2}{r^2}.$$

The equation you just derived using Newtonian physics also turns out to be the equation that describes an expanding universe in general relativity. In this section you'll explore some of the implications of this equation.

Write your answers here:

14.5.1 Discovery Exercise: Rotation Curves

The Milky Way galaxy is a wide disk of stars orbiting a dense, central bulge. For simplicity we'll assume here that essentially all of the mass M of the galaxy is in the bulge and that the stars outside the bulge are in circular orbits.[12]

With those assumptions, use Newton's second law and Newton's law of gravity to calculate the orbital speed of a star outside the bulge, as a function of its distance from the center of the galaxy. Make a sketch of your calculated function $v(r)$.

You'll see in this section that the observed function $v(r)$ for essentially all galaxies is very different from what you just calculated. That discrepancy between prediction and observation led to the discovery of "dark matter," one of the two largest contributors to the overall energy of the observable universe.

12 The second assumption is reasonable. The first one is not – less than half the observed mass in the galaxy is in the central bulge – but it will work to make the qualitative point we want to illustrate here.

Write your answer here:

14.6.1 Discovery Exercise: Problems with the Big Bang Model

When the universe dropped below the Planck density, as far as we know it could have had any value of curvature. But we can constrain that value based on our current observations.

For simplicity, assume throughout this exercise that the universe has been matter-dominated from the beginning through today.

1. If the universe today is 13.8 billion years old, by what factor has the scale factor increased since it was one second old?

   See Check Yourself #27 at www.cambridge.org/felder-modernphysics/checkyourself

2. We observe that today the density term in the first Friedmann equation is at least 100 times larger than the curvature term. How much larger must the density term have been at $t = 1$ s?

Write your answers here:

14.7.1 Discovery Exercise: Inflation and the Very Early Universe

We have discussed the remarkable fact that dark energy maintains a constant energy density (measured, for instance, in J/m^3) as the universe expands. If you worked through Problem 8 in Section 14.6, you have seen that the prevalence of dark energy is causing the curvature of the universe to decrease, but not enough to explain the extremely low curvature we measure today.

But what if a type of energy with that same property had been around just after the universe dropped below the Planck density?

1. As you may recall, the first Friedmann equation with a constant energy density and negligible curvature leads to the growth equation $a = a_0 e^{Ht}$, where $H = \sqrt{8\pi G\rho/(3c^2)}$. If the energy density were the Planck density (4.6×10^{113} J/m^3) and the universe expanded in this way for 10^{-35} s, by what factor would the scale factor increase?

   See Check Yourself #28 at www.cambridge.org/felder-modernphysics/checkyourself

2. Suppose the ρ term in Equation (14.1) was 10 times bigger than the curvature term at the start of the 10^{-35} s expansion you just calculated. During that period, ρ stayed constant while a increased. By what factor would the ρ term end up bigger at the end of that expansion?

Write your answers here:

Discovery Exercise: The Electric Force and the Gravitational Force

The questions in this Discovery Exercise are based on purely classical mechanics: no relativity or quantum mechanical ideas are required.

You are attempting to predict the motion of a proton and an electron. The two particles are far apart, and exert no significant force on each other. Note that we're not asking you to actually do any calculations or write any formulas below; we are asking questions about how you *would* do some calculations. If the whole thing takes you more than five minutes, you're probably overthinking it.

In your first experiment, the two particles are immersed in a constant electric field.

1. First you calculate the force that the electric field exerts on the proton, and also the force it exerts on the electron. What *property* of each particle do you need to know, in order to calculate these forces?
2. From those forces, you calculate the resulting acceleration of each particle. What property of the proton and electron do you need to know for this step?
3. Briefly explain why the electron will accelerate much more than the proton, based on both steps above.

In your second experiment, the two particles are immersed in a constant *gravitational* field.

4. Once again, you begin by calculating the two forces exerted by this field. What property of the two particles do you need to know in order to calculate these forces?
5. From those forces, you calculate the resulting accelerations. What property of the two particles do you need to know for this step?
6. This exercise has been designed to point out a key difference between the classical electric force and the classical gravitational force. What point do you think we're driving at?

Write your answers here: