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Active Reading Exercise: A Potential Step with $E > U_0$

1. Write the energy eigenstate at $x \geq 0$ for a particle with $E > U_0$.
2. Your answer to Part 1 includes two pieces. What does each of the two pieces represent about our particle?
3. Neither of these two pieces blows up as $x \to \infty$, but one of them has to equal zero anyway. Can you guess which one, on physical grounds?

Active Reading Exercise: Galileo's Spaceship

Imagine yourself on a spaceship far outside our solar system. It's a roomy ship with all the comforts of home, but it has no windows and no communication of any kind with the outside world. You hold your hand out in the center of the room with a ball in it – not on a table or the floor, just in mid-air – and then you open your hand and let go of the ball.

1. What does the ball do if the ship suddenly speeds up?
2. What does the ball do if the ship suddenly slows down?
3. What does the ball do if the ship makes a hard right turn?
4. What does the ball do if the ship stays the course, moving incredibly fast without change?

Jot down all four answers before you continue reading!

Active Reading Exercise: The Airplane and the Mountains, Part 1

Imagine our airplane-and-mountains scenario above with one change: instead of a flashlight shining straight up from the floor, a gun fires straight up from the floor. The bullet, like the light beam, travels at constant speed to the ceiling. (We don't want to think about what happens after that.) As with the light beam, assume the bullet reaches the ceiling exactly as the plane passes the second mountain. You can assume that the effect of gravity on the bullet is negligible during its flight.

1. Draw the path of the bullet twice: first in the plane reference frame, and then in the mountain reference frame. (They should not look the same!)
2. How far does the bullet travel in the reference frame of an observer in the airplane?
3. How far does the bullet travel in the reference frame of an observer on the mountains?
4. Based on purely Newtonian/Galilean assumptions, the two observers calculate different speeds. Which observer sees the bullet going faster? Briefly explain your answer.

Don't read on until you have written down answers to these questions. It's okay if you get some wrong, but commit to your best shot before reading our answers.

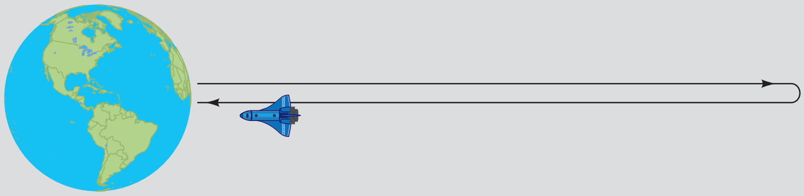
Active Reading Exercise: The Airplane and the Mountains, Part 2

Suppose the mountain clocks say one second elapsed between the two events (light-emitted and light-hits-ceiling). Does the plane clock say that it took more than one second, or less than one second?

Don't just write down an answer. Write down a sentence or two explaining how your answer allows the two frames to agree on the speed of light.

Active Reading Exercise: The Twin Paradox

Emma goes on a long voyage in a near-light-speed rocket ship. She travels in a straight line to her destination, turns around (more or less instantaneously), and returns to Earth. Her twin, Asher, has been waiting for her in California the whole time.



Just before Emma steps out of her rocket, Asher thinks: "Because she has been moving the whole time, her clocks were running slowly. So she has aged less than I have." Meanwhile, Emma thinks: "From my point of view Asher was moving this whole time so his clock was running slowly. When I get home he will have aged less than I have."

Question: Who was right, and what was wrong with the other one's argument?

Don't keep reading before you think about this one a while. As the two twins stand side-by-side at the end, they *must* agree about which twin has aged more. Does this paradox prove that Einstein's theory is logically inconsistent?

Hint: Time dilation is a result of Einstein's two postulates. Look back at p. 12 (of the main text) and re-read them. The answer is there!

Active Reading Exercise: Coordinate Time, Proper Time

An airplane flies by Mountain A, and then later flies by Mountain B.

1. The clock on the airplane registers a certain time between these two events. Is that a proper time, a coordinate time, or both?
2. The perfectly synchronized clocks on the two mountains also register a certain time between these two events. Is that a proper time, a coordinate time, or both?

Active Reading Exercise: Length Contraction

In the mountain frame, the distance between Mountains A and B is L. In the airplane frame, is the distance between the mountains greater than L, or smaller than L?

Active Reading Exercise: The Plane Gazes Back

When the airplane crew look at the airplane clock, they see it running at a perfectly normal speed. When the mountain dwellers look at a mountain-based clock, they likewise see it running normally.

But when the mountain folk look at the clock on the airplane, they see it running too slowly. From their perspective, the airplane clock advances by less than one second when a second goes by.

Question: What do the airplane riders see when they look at the clock on a mountain? Do they see it running too slowly, too quickly, or at normal speed?

As always, jot down your answer, along with a brief justification, before you read ours.

Active Reading Exercise: The Other Clock

In the reference frame of the airplane, what did the clock on Mountain A read at the moment when the airplane reached Mountain B? Jot down your answer along with a brief explanation.

Active Reading Exercise: The Position

Let x' be the blue junction at the event with R coordinates x and t. Each of the questions below is easiest to answer by considering how the blue (R$'$) rulers are behaving from the point of view of Frame R.

1. How far does the x' junction travel from $t = 0$ to the event at time t?
2. Knowing that, at time t, the x' junction reached position x, where was it at time 0?
3. So now that you know *where* it was at $t = 0$, how many rulers are between $x' = 0$ and that junction?

Active Reading Exercise: Spacetime Diagrams

Draw a spacetime diagram with world lines for the following three objects. Assume all three start at $x = 1$ at time 0.

1. An object at rest.
2. An object moving in the negative direction at a constant velocity.
3. An object moving in the positive direction and slowing down as it goes.

Make all three sketches before reading further!

Active Reading Exercise: Axes on Spacetime Diagrams

1. Draw the axes of a spacetime diagram. The coordinates x and t are of course defined in some reference frame R.

Now consider a frame R′ moving at speed $c/2$ to the right, relative to R.

2. Draw the t' axis on your Frame R spacetime diagram: that is, all the points for which $x' = 0$.
3. Add the x' axis into the same drawing.

Will either axis be horizontal, or vertical? Will they be perpendicular to each other? None of these answers is obvious without some thought. Try to go back and forth between two modes of thinking: purely visual (think about the twin story above), and algebraic (the Lorentz transformations are in Appendix B).

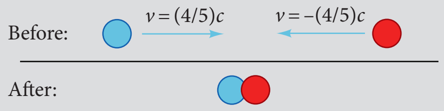
Active Reading Exercise: Proper Time and Spacetime Interval

Figure 2.4 shows the world lines of Emma (blue) and Asher (red). *The five items below should take less than* 30 *seconds of calculation each. If it gets messier than that, try a different approach.*

1. Write the spacetime interval for the first (outgoing) leg of Emma's trip. Your answer will be a function of the constants T and L.
2. Write Emma's proper time for that same leg of the journey.
3. Write Emma's proper time for the second (incoming) leg of the journey.
4. Write Emma's proper time for the entire journey.
5. Is the final result greater than, or less than, Asher's proper time $2T$?

Active Reading Exercise: An Inelastic Collision

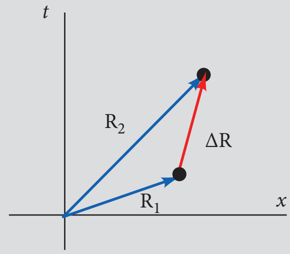
Two balls of putty, each of mass m, are moving toward each other. Before they collide, each ball is moving with speed $(4/5)c$ in the lab frame. After they collide, they stick together in one lump.



1. How will the lump move after the collision? (This is not a trick question; the answer should be obvious.)
2. Use Equation (2.2) to calculate the energy before and after the collision. Did the two come out equal?

Active Reading Exercise: Four-Displacement

The figure shows the journey of an object from the event represented by four-vector $\mathbf{R}_1$ to the event represented by four-vector $\mathbf{R}_2$.



1. What does the horizontal component of $\Delta\mathbf{R}$ tell us about the journey?
2. What does the vertical component of $\Delta\mathbf{R}$ tell us about the journey?
3. What does the slope of $\Delta\mathbf{R}$ tell us about the journey?

Active Reading Exercise: Four-Vectors and Scalars

On p. 85 (of the main text) we made three specific points about rotating axes: a vector transforms according to Equations (2.6), a scalar remains invariant, and the magnitude of a vector (defined as $\sqrt{x^2 + y^2}$) is invariant (a scalar).

Before reading further, can you figure out the three equivalent points about four-vectors and inertial reference frames? What equations are used to translate a four-vector such as (ct, x, y, z) to a different inertial reference frame? What formula, involving the components of a four-vector, remains invariant when you do so?

Active Reading Exercise: What is a Scalar?

We define a scalar as a quantity that is the same according to all inertial reference frames. A simple example is any counting number: how many there are of something. If I go to the store and buy a box of marbles, everyone will agree that the box has 20 marbles in it. Another scalar is an object's temperature.

List at least two quantities that are scalars (besides counting numbers and temperature) and at least two that aren't.

Active Reading Exercise: Δt and $\Delta\tau$

We're describing a scenario in which, in your reference frame, an object moves at constant velocity for a time Δt, and its own clock ticks off $\Delta\tau$ in that time. Write an equation relating the two.

Active Reading Exercise: The Direction of Four-Momentum

If you were to draw the object's four-momentum on the spacetime diagram in Figure 2.18, where would it point? Parallel to $\Delta\mathbf{R}$, above it, or below it?

Active Reading Exercise: The Speed of Sound

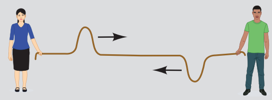
Imagine that you are on an airplane traveling at 100 m/s through still air. The airplane emits a sound that travels 300 m/s in every direction, relative to the air. Using Galilean relativity (or common sense), and ignoring Einsteinian relativity (which is not relevant at these speeds anyway), answer the following questions from your perspective on the airplane.

1. How fast do you measure the sound moving forward – that is, moving in the same direction the airplane is traveling?
2. How fast do you measure the sound moving in the opposite direction?

Active Reading Exercise: The Third "Alice's Rope" Scenario

Did you follow what happened when Alice's and Bob's bumps collided? Let's find out by changing the story just a bit.

- *Alice does her same signature shake. But this time Bob gives the rope a downward shake, causing an upside-down bump to move toward Alice.*

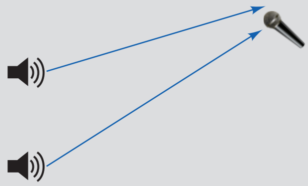


1. Draw and/or describe what the rope will look like when the two bumps meet in the middle.
2. Draw and/or describe what the rope will look like a second or two after that.

Don't read further until you have written down both guesses!

Active Reading Exercise: What Does the Microphone Hear?

Consider two speakers emitting sinusoidal sound waves that are identical in every way: same amplitude, same frequency, and perfectly in phase (meaning the speakers peak at the same time). A microphone is placed nearby.



What will the microphone pick up if . . .

1. the microphone's distance to the two speakers is the same?
2. the microphone is exactly one wavelength closer to one speaker than the other?
3. the microphone is exactly half a wavelength closer to one speaker than the other?

Write down your answers to all three questions before reading further.

Active Reading Exercise: The Single-Slit Scenario

Draw two graphs. One graph represents bullet-hole density along Wall B in the first box, and the other graph represents wave height along Wall B in the second box. These graphs will be qualitative (we haven't given you any numbers to work with), but pay particular attention to how the two shapes are similar and how they differ.

Do not continue reading until you have drawn the graphs!

Active Reading Exercise: The One-at-a-Time Double-Slit

Imagine the double-slit experiment with a one-photon-at-a-time light source. You fire the first photon and a dot appears at one point on Wall B. You fire a second photon, and Wall B now has two dots. Then a third and a fourth. Eventually you will have so many dots that you can plot their density on Wall B.

Question: Will you see a "particle-like" pattern, with high density behind each slit and lower as you move away? Or will you see the alternating bands of high and low density that we associate with a wave?

Formulate your own answer to that question. Write it down, along with a few sentences explaining your reasoning. Don't be afraid to be wrong! But don't just guess haphazardly either; think it through.

Do not read further until you have written down your best guess.

Active Reading Exercise: The Orthodox Interpretation

Re-read the description above of the orthodox interpretation of quantum mechanics. Make sure you see how this model explains the alternating bands in the double-slit experiment.

Then make a list of things that don't fully make sense. You may want to break your list into two categories: *questions* about the orthodox interpretation, and *objections* you might raise. (If it helps, think of yourself as Einstein, who wrote long letters full of vehement objections to Bohr.) Think about the mathematics of waves, and the mathematics of probability. Think about particles and measurements. Think about alternative explanations.

> *Those who are not shocked when they first come across quantum theory cannot possibly have understood it.*
>
> – Niels Bohr

Active Reading Exercise: Double-Slit Variations

Read the first scenario below and *write down* your best prediction of the pattern you'll see on the back wall. Then check it against our online answer. Then do the same for the second scenario, and then the third.

1. Run the double-slit experiment one photon at a time, but put detectors at both slits so you can measure a photon going through the left slit, the right slit, or both simultaneously.
2. The same as above, but with no detector in the right slit. (If the detector goes off, a photon went through the left slit. If something appears on Wall B but the detector didn't go off, the photon passed through the right slit.)
3. Repeat the original experiment (no detectors) but emit electrons – still one at a time – instead of light.

Active Reading Exercise: The Ultraviolet Catastrophe

In the preceding Example we used a made-up $E_w(\nu)$ function to predict the spectrum $S(\nu)$ and the energy density ρ inside a cavity. Now you use the classically predicted $E_w(\nu)$ (Equation (3.6)) to calculate the same variables. (This calculation is easier than the one we did in the Example.) Write down your answers before you look at ours. Can you see why this result was called a "catastrophe"?

Active Reading Exercise: Tweaking ΔE

The traditional calculation of E_w is an integral, the limit of the Table 3.1 as $\Delta E \rightarrow 0$. Planck decided to use sums instead.

1. Suppose Planck's goal was to achieve a very low value for E_w. Should he choose a high ΔE, or a low one? How low could he get E_w to go?
2. Suppose Planck's goal was to achieve a very *high* value for E_w. Now what kind of ΔE should he choose? How high could he make E_w?
3. What did we say (earlier) that Planck's goal actually was?

Active Reading Exercise: Photoelectric Predictions in the Classical Model

Imagine setting up the photoelectric effect and measuring the current of electrons flowing out of Plate B. Based on the classical model of light in the paragraphs above . . .

1. How would an increase in the intensity (or amplitude) of the light – holding the frequency constant – affect the time lag, the final current, and the maximum kinetic energy of released electrons?
2. How would an increase in the frequency of the light – holding the amplitude constant – affect the time lag, the final current, and the maximum kinetic energy of released electrons?

Your answers can be brief, but don't read further until you have written them down!

Active Reading Exercise: A Photon–Electron Collision

An unbound electron is initially at rest when a photon of frequency ν collides with it. As in most collisions, the electron and photon move off in different directions afterward. Would the frequency of the photon after the collision be lower, the same, or higher than it was before the collision? Even if you're not sure, write down an answer and a justification for it before reading on.

Active Reading Exercise: Quantization in the Bohr Model

Bohr's model explicitly says that the angular momentum of the electron orbits is quantized. That assumption implies that some other properties of the orbits must be quantized as well. Name two other properties and explain how quantization of angular momentum (for circular orbits) implies that they must also be quantized.

Don't read further until you have written down at least two responses.

Active Reading Exercise: Matter Waves

Based on what you've learned so far about photons, how would you design an experiment to test de Broglie's hypothesis of matter waves? Try to write down at least one answer before reading on.

Active Reading Exercise: Three Probabilities

In our simple three-point universe, why would it be impossible for the particle to have the wavefunction $\psi(1) = \psi(2) = \psi(3) = 1/2$?

Active Reading Exercise: Repeated Measurements

A particle has the wavefunction we showed in the Example on p. 194. You measure its position and find it very near $x = 3L/4$. If you then immediately measure its position again, is the probability of finding it near that same spot the second time higher than, lower than, or the same as it was before the first measurement?

Active Reading Exercise: Discrete Expectation Value

Earlier we considered a particle that could be at $x = 1$ with probability $1/9$, or at $x = 2$ with probability $4/9$, or at $x = 3$ with probability $4/9$. Suppose you took nine such particles, measured each of their positions, and then averaged the results. What would you expect to find?

Before reading further, do that calculation. Then see if you can generalize it to a formula.

Active Reading Exercise: Position Uncertainty

Suppose you find that a wavefunction has $\langle x \rangle = 20\,\text{Å}$, with $\Delta x = 3\,\text{Å}$. What do you conclude?

Active Reading Exercise: Slightly Modified Uncertainty

Draw a quick sketch of a wavefunction that is very similar to Wavefunction B in Figure 4.12, but with a slightly decreased position uncertainty Δx. How do you think your change affected the momentum uncertainty Δp?

Make your sketch and write your answer before going on. We'll give our answer after we present the uncertainty principle.

Active Reading Exercise: Deviations from the Average

All of these questions refer to the gradebook, Table 4.1 on p. 206 of the main text.

1. For each student calculate ΔG, the student's score minus the class average.
2. Find the average value of all the ΔG scores.
3. Why is the average of ΔG not a good measure of how spread out the class scores are?

Active Reading Exercise: Motion in a Potential Well

The left-hand object in Figure 5.1 is released at rest at the position marked $x = a$. How will it move initially? How will it move in the long run? As you describe its motion, describe its mixture of kinetic and potential energy at different times.

Active Reading Exercise: Energy Eigenstates

After analyzing a particular system, you have determined that two of its energy eigenstates and eigenvalues are

$$\psi_1(x) = Ae^{-kx^2} \text{ with associated eigenvalue } E_0$$
$$\psi_2(x) = Be^{-2kx^2} \text{ with associated eigenvalue } 4E_0$$

1. If the system is known to be in the state $\psi(x) = Ae^{-kx^2}$ when you measure its energy, what energy will you find?
2. If the system is known to be in the state $\psi(x) = (1/2)Ae^{-kx^2} + (\sqrt{3}/2)Be^{-2kx^2}$, what two possible energies might you find, and with what probabilities? (We haven't told you how to answer this part, but make your best guess before reading on. You may be able to guess at least part of the answer.)

Active Reading Exercise: A 1D Particle in a Box

A particle can move with no forces in the region $0 < x < L$, but cannot move anywhere outside that region.

1. What must be true of the potential energy in that region, based on the fact that the particle feels no forces?
2. What must be true of the potential energy at the boundaries in order to prevent the particle from leaving that region?
3. Assuming the particle is purely classical and it begins at $x = L/2$ moving to the right, describe its behavior over time.

Active Reading Exercise: Allowed Energies in an Infinite Square Well

What values of the constant E satisfy Equation (5.6)? This is a purely mathematical question, and it has infinitely many correct answers. If you're stuck, start by writing down one possible non-zero value of E that works. Then see if you can write a general formula for all of them.

Active Reading Exercise: What Did We Find?

Equation (5.8) tells us the energy eigenstates of a particle in our infinite square well. Based on that:

1. If a particle is in the following state, what is its energy?

$$\psi_A(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{4\pi}{L}x\right)$$

2. If a particle is in the following state, what energies might a measurement find, and with what probabilities?

$$\psi_B(x) = \sqrt{\frac{1}{8}}\left[\sqrt{\frac{2}{L}} \sin\left(\frac{4\pi}{L}x\right)\right] + \sqrt{\frac{7}{8}}\left[\sqrt{\frac{2}{L}} \sin\left(\frac{5\pi}{L}x\right)\right] \tag{5.9}$$

Active Reading Exercise: A Simple Harmonic Oscillator Solution

1. Why is the wavefunction in Figure 5.8 not physically possible?
2. What does that tell us about the energy eigenstates of the simple harmonic oscillator?

Active Reading Exercise: The Finite Square Well

1. What physical requirements can we use to determine the unknown constants in Equation (5.15)?
2. Use those requirements to write down four equations involving the constants A, B, D, F, and E (the energy). Some of your equations should have L in them, but none of them should have the variable x.

Active Reading Exercise: Evolution of an Energy Eigenstate

Consider a particle in an infinite square well, whose initial wavefunction is given by the following energy eigenstate:

$$\Psi(x,0) = \psi_3(x) = \sqrt{\frac{2}{L}}\sin\left(\frac{3\pi}{L}x\right), \qquad E_3 = \frac{9\hbar^2\pi^2}{2mL^2}.$$

1. Write $|\Psi(x,0)|^2$, which determines position probabilities at $t = 0$.
2. Write $\Psi\,(x,\pi/3)$, the wavefunction at $t = \pi/3$, based on Equation (5.20).
3. Write $|\Psi(x,\pi/3)|^2$, which determines position probabilities at $t = \pi/3$. Simplify your answer.
4. How did the position probabilities change between these two times?

Active Reading Exercise: Evolution of a Sum of Two Eigenstates

As an example of our two-eigenstate rule, suppose a particle starts out at $t = 0$ as an equal combination of two real-valued energy eigenstates ψ_1 and ψ_2:

$$\psi(x) = A\left[\psi_1(x) + \psi_2(x)\right]. \tag{5.21}$$

Will the probability position function for $\Psi(x,t)$ change over time? Don't just say yes or no: calculate the position probability distribution at a time $t > 0$ and see whether our result for the single eigenstate is still valid or not.

Active Reading Exercise: Energy Probabilities

In Section 5.2 we said that if a particle is in the state $\psi(x) = A\psi_1(x) + B\psi_2(x)$ and you measure its energy you will have probability $|A|^2$ of getting E_1 and $|B|^2$ of getting E_2. If you let the particle evolve forward until some later time t and then measure its energy, what is the probability you'll get the answer E_1?

Active Reading Exercise: Standing Wave Wavelength and Period

A wave initially described by the function $y = 10\sin(4x)$ oscillates as described above: it shrinks until it reaches the x axis, grows until it reaches its original size on the other side of the x axis, and then oscillates back over and over.

Warning: We're about to ask you three questions. One of the three is a trick question!

1. What is the highest y-value ever reached at any place or time?
2. The "wavelength" is the distance along the x axis from one crest to the next. What is the wavelength of this wave?
3. The "period" is the time it takes for a peak to go all the way down to a valley and then back up to the peak. What is the period of this oscillation?

Don't read further until you have written down all three guesses. Then, if you want to take one more adventurous step, see if you can write down the mathematical formula that describes this vibrating string as a function of both space and time. (You will have to introduce a new constant whose value you don't yet know.)

Active Reading Exercise: The Standing Wave Equation

Equation (6.1) represents a "standing wave." Suppose it gives the height of a vibrating string.

1. Imagine taking a picture of this vibrating string at $t = T/4$. The resulting snapshot is a sine wave in x. What are its amplitude and wavelength?
2. Now imagine watching the y-value at $x = \lambda/8$ moving up and down. The motion of that point is a sine wave in t. What are its amplitude and period?
3. Now, can you put all that together to see a mental video of the standing wave?

Active Reading Exercise: Traveling Wave

Suppose a wave initially equal to $y = 5\sin(\pi x)$ moves to the right at speed 6.

1. If you take a snapshot of this wave – that is, look at the function $y(x)$ at a particular t-value – you see a sine wave. What are its amplitude and wavelength?
2. If you stand at one particular x-value and watch the y-value over time, you see a sinusoidal oscillation that could be described by a $y(t)$ function. What are its amplitude and period? (Unlike the Exercise on p. 262, this time we have given you enough information to determine the period.)
3. What function $y(x, t)$ describes this traveling wave?

Don't read further until you have written down all three answers!

Active Reading Exercise: A Free Particle

If you haven't done it yet, do Parts 1 and 2 of Discovery Exercise 6.2.1.

Active Reading Exercise: Momentum of a Cosine

Suppose that a particle's wavefunction is $\psi(x) = A\cos(kx)$ for some constants A and k.

For the moment, ignore everything we've said in this chapter: eigenstates, normalizability, all of it. Instead, go back to the de Broglie relation $\lambda = h/p$ (which was, as you recall, one of the elements of the old quantum theory that was baked into real quantum mechanics). What does the de Broglie relation tell us about the momentum of this particle?

Active Reading Exercise: Narrow and Wide Bumps

The Example on p. 281 shows a wavefunction that is a bump centered on $x = 0$, and its Fourier transform (which is proportional to its momentum probability distribution). If we had made the wavefunction bump twice as wide, how would that have changed the momentum probabilities?

Hint: Look at how L appears on the horizontal axes of $\psi(x)$ and of $\hat{\psi}(k)$.

Active Reading Exercise: $y''(x) = k^2 y(x)$

1. Find a real non-zero function that satisfies the differential equation

$$\frac{d^2y}{dx^2} = y.$$

   Don't try to apply a technique or do any algebra here; just think of a real function that is its *own second derivative*. *Hint*: You won't be able to achieve this with sines or cosines, since we're not allowing i in your solution.

2. Find a *different* real non-zero solution that is not a multiple of your first solution.

3. Write the general solution, with two arbitrary constants, to the following differential equation:

$$\frac{d^2y}{dx^2} = 9y.$$

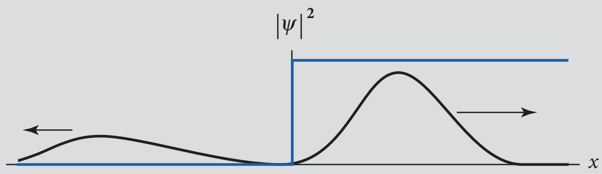
Active Reading Exercise: A Potential Step with $E > U_0$

1. Write the energy eigenstate at $x \geq 0$ for a particle with $E > U_0$.
2. Your answer to Part 1 includes two pieces. What does each of the two pieces represent about our particle?
3. Neither of these two pieces blows up as $x \to \infty$, but one of them has to equal zero anyway. Can you guess which one, on physical grounds?

Active Reading Exercise: Interpreting the Solutions for a Potential Step

A particle with energy $E > U_0$ is incident on a potential step of height U_0. Here's the formula for an energy eigenstate, and a picture of the wavefunction a while after it hits the barrier. The parts of the wavefunction with coefficients A, B, and C represent the incident wave, reflected wave, and transmitted wave, respectively.

$$\psi(x) = \begin{cases} Ae^{i\left(\sqrt{2mE}/\hbar\right)x} + Be^{-i\left(\sqrt{2mE}/\hbar\right)x} & x < 0 \\ Ce^{i\left(\sqrt{2m(E-U_0)}/\hbar\right)x} & x \geq 0 \end{cases}$$



1. How can we keep talking about "the energy of the particle" and writing things like "$E > U_0$" when the particle's wavefunction is an integral over infinitely many eigenstates, each representing a different energy? An unbound particle can never be in a state of definite energy!
2. Is the ratio $|B/A|$ the same for every eigenstate?
3. Does the ratio $|B/A|$ stay the same as the wave evolves through time?

Active Reading Exercise: Plugging in the Separable Solution

If $\psi(x)$ is a function of x (but not of t), and $T(t)$ is a function of t (but not of x),

1. What is $\dfrac{\partial}{\partial t}[\psi(x)T(t)]$?
2. What is $\dfrac{\partial^2}{\partial x^2}[\psi(x)T(t)]$?
3. Bearing those answers in mind, plug the guess (Equation (6.20)) into the PDE (Equation (6.19)).

Active Reading Exercise: The Time Equation

Equation (6.23) can be rewritten as

$$\frac{dT}{dt} = -\frac{iE}{\hbar}T(t).$$

(Note that, as usual, we rewrote $1/i$ on the right side as $-i$.)

1. Solve that equation. You can do this by using separation of variables (not the PDE technique we just taught you, but the ODE technique you may have learned in previous math courses). You may also be able to solve it just by thinking about it.
2. Equation (6.22) became a familiar result, the time-independent Schrödinger equation. What familiar conclusion does Equation (6.23) lead to?

Active Reading Exercise: The Equations for *Y* and *Z*

Separate variables to turn Equation (7.10) into two ordinary differential equations, one for $Y(y)$ and one for $Z(z)$. Both equations will have a new constant p^2 that you'll introduce in the same way we introduced k^2 above. (One of the equations will also still have k^2 in it.)

Active Reading Exercise: Covering All of Space with Spherical Coordinates

In Cartesian coordinates you cover all of space by letting x, y, and z range from $-\infty$ to ∞. What range of values do r, θ, and ϕ need in order to cover all of space? Make sure that you try to answer this yourself before reading on, but we will start you off with the hint that none of the three variables needs to go from $-\infty$ to ∞.

Active Reading Exercise: The ϕ Probability Distribution

For any values of the variables m_l and ϕ, it is always the case that $|\Phi(\phi)| = \left|e^{im_l\phi}\right| = 1$.

1. What does that fact tell us about the ϕ dependence of position probabilities for an electron that is in an eigenstate?
2. What does that same fact tell us about the ϕ dependence of position probabilities for an electron that is in a superposition of different eigenstates?

Active Reading Exercise: The Stern–Gerlach Experiment

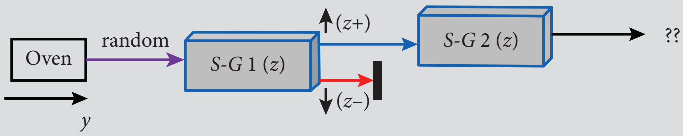
Imagine a Stern–Gerlach experiment with hydrogen atoms. Each atom is bent up or down by an amount proportional to the z angular momentum of its electron. Then each atom strikes a detector on the other side of the apparatus and makes a mark where it hits. Draw what you would expect the pattern on the detector to look like in each of the following cases:

1. A classical model, in which the electron in each atom is orbiting with a random orientation.
2. The Bohr–Sommerfeld model, in which the electron in each atom can have $L_z = \pm\hbar$.
3. A quantum mechanical hydrogen atom in the ground state.

Active Reading Exercise: Two Identical Spin Measurements

A stream of silver atoms heading in the y direction passes through a Stern–Gerlach apparatus that separates it into two separate streams, one ↑ and one ↓. The initial stream was random, so the atoms split roughly 50/50 between the two outgoing streams.

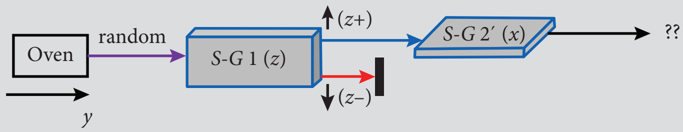
Now the ↑ stream passes through a second apparatus identical to the first. It splits *that* stream based on positive and negative S_z-values.



What comes out of the second device? (This is *not* a trick question.)

Active Reading Exercise: Two Orthogonal Spin Measurements

We begin once again with an apparatus that separates atoms into two streams based on S_z. But this time the ↑ stream goes into an apparatus that redirects atoms based on S_x, the x components of their spins.

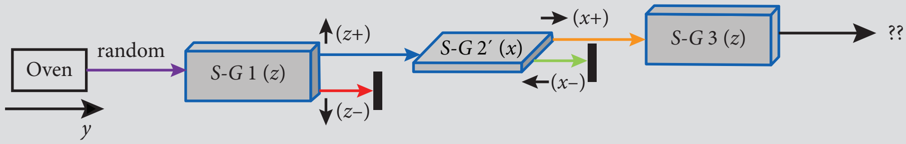


What comes out of the second device?

Active Reading Exercise: Three Alternating Spin Measurements

We begin once again by segregating atoms into two streams based on S_z. Then the ↑ stream goes into the S_x-splitter as before, and comes out 50/50.

But now we focus on one of the two streams coming from that second device. Every atom in this stream has now been sorted twice: first based on having a positive S_z, and then by having a positive S_x. We feed this stream into a third Stern–Gerlach apparatus; this one – like the first – sorts the stream on the basis of the z component of spin.

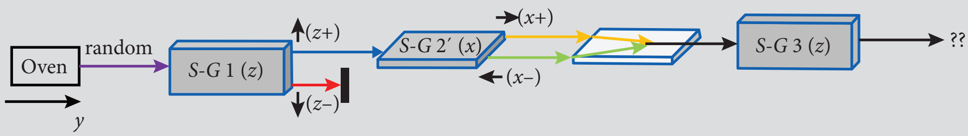


What comes out of the third device?

Active Reading Exercise: Recombining the Streams

We use a Stern–Gerlach apparatus to produce a stream of atoms known to have positive S_z. That is then sorted into two streams, one with positive S_x and one negative. We know that if we measured either of those two streams for its S_z-values, the results would come out 50/50.

But instead we *recombine* the two streams before performing any further measurements. There is no way to look at an atom in the resulting stream and determine whether it came from the → stream or the ← one.



What will we find if we now measure S_z for this final stream of atoms?

Active Reading Exercise: The Zeeman Effect

Suppose you put a gas of atoms in a magnetic field pointing in the positive z direction ($\vec{B} = B\hat{k}$).

1. Would the external magnetic field cause the energy of the eigenstates to depend on l, on m_l, or on both?
2. For whichever one(s) you chose, would the energy be higher when that quantum number was high or low?

Ignore spin for now.

Active Reading Exercise: Electron Screening

Jot down your answers to the following questions before reading on. (If you're stuck on one of them you should still try to answer the other; they are independent of each other.)

1. We said above that the electron screening model predicts an ionization energy of 3.4 eV for lithium, but that model isn't exact. Would you expect the actual value to be higher or lower than that? Why?
2. What does the electron screening model predict for the effective charge felt by the outer electron of beryllium ($Z = 4$)?

Active Reading Exercise: Subshell Energies

A lithium atom has its $n = 1$ shell filled by the first two electrons. The third electron will be either in subshell $2s$ or in $2p$.

1. Based on Figure 8.3, which of these two subshells has a higher probability of the electron being found at $r < a_0$?
2. Explain how your answer to Part 1 tells us which subshell has lower energy (and will therefore be occupied first). *Hint*: Your answer should include some form of the verb "to screen."

Active Reading Exercise: Ionization Energy

Argon ($Z = 18$) in its ground state has all of the states up through $n = 3$ filled. Potassium ($Z = 19$) has all of those states plus one $4s$ state filled, and calcium ($Z = 20$) has the same plus a second $4s$ electron. Rank these three atoms in order from lowest ionization energy to highest. You do not need to do any calculations for this problem. Just explain in words which ones you would expect to be higher and lower, and why.

Active Reading Exercise: Electron Affinity

Fluorine ($Z = 9$) in its ground state has five of the six $2p$ states filled. Neon ($Z = 10$) has all six of those states filled.

1. If you were to add an electron to each of these two atoms, which one would release more energy and why?
2. Therefore, which of these two elements is more likely to bond with another element by absorbing an electron?

Active Reading Exercise: The K_β Plot

Suppose you could do an experiment using many different elements (aka many different Z-values), but all of them undergo K_α transitions only. Based on Equation (8.2), what would be the shape of your $\sqrt{\nu}$ vs. Z plot? You can start your answer with just one word describing the shape, but then you can make several other predictions about the details of the graph.

Active Reading Exercise: The Proton and the Hydrogen Atom

When the proton and hydrogen atom get closer together (but not yet so close that the lone proton is inside the electron cloud), there *is* a net force between them. Which way (attractive or repulsive) and why? *Hint*: Try sketching how our picture of the hydrogen atom will change as it gets close to the proton.

Active Reading Exercise: Sum and Difference Wavefunctions

Consider two protons close enough to each other that ψ_1 and ψ_2 overlap. Sketch ψ_S and ψ_D, similarly to how we did above for the case of two widely separated protons. In which state is the electron more likely to be found close to the midpoint between the two protons?

Active Reading Exercise: F_2

Two individual fluorine atoms have, between them, ten $2p$ electrons. How will those ten electrons fill the available states in the two-proton system?

Active Reading Exercise: The Not-Quite-Equilibrium Radius

Two atoms in perfect bonded equilibrium are jostled. They are now a bit too far apart (and are therefore pulled toward each other), or else a bit too close together (and are therefore pushed apart). What do they do now? *Hint*: "They come to rest at their equilibrium radius" is a nice, simple answer that doesn't conserve energy.

Active Reading Exercise: Microstates of a Paramagnet

Suppose you have a paramagnet with four atoms. One possible microstate is UUDD, meaning the first two are up and the last two are down.

1. How many possible microstates are there? (There are several ways to figure this out, but if you're stuck you can just list them all.)
2. How many microstates have all four atoms pointing up?
3. How many microstates have exactly two atoms pointing up?
4. If you were to measure the total magnetic field of this paramagnet at some instant, what is the probability that you would find all four atoms pointing up? What is the probability that you would find exactly two atoms pointing up (i.e. zero magnetic field)?

Active Reading Exercise: Identifying Our Assumption

We calculated above that a 4-atom paramagnet has 16 possible microstates, and that 6 of those involve equal numbers of up and down atoms. From those numbers we concluded that the probability of the system being in a state with equal numbers of up and down atoms was 6/16.

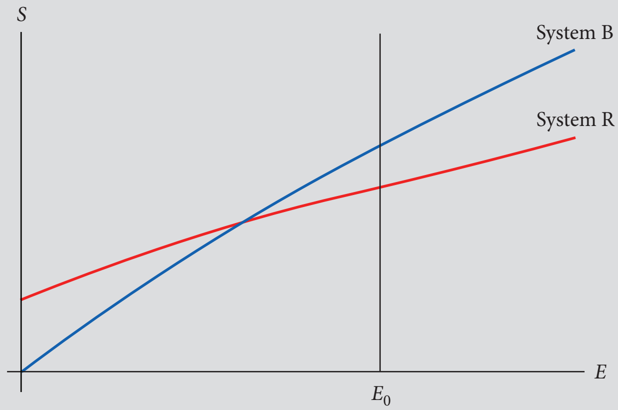
What assumption did we have to make to go from those two numbers to that conclusion about probability? (We're not asking what assumptions went into finding the numbers 16 and 6; that was just counting. We are asking what assumption went into the next step, where we said that their ratio equals the probability of finding that particular macrostate.)

Active Reading Exercise: An Evolving Paramagnet

Suppose the five-atom paramagnet from the previous example begins with all five atoms pointing down. But every millisecond, each atom has a 1% chance of randomly flipping its state. What do you expect to see after 5 ms? What do you expect to see after a year? (We are asking for some thought and a general answer; we are not asking for calculations.)

Active Reading Exercise: Entropy vs. Energy

Two systems called "System B" and "System R" are in contact, so they can exchange energy with each other. The blue curve below shows the entropy of System B as a function of its total energy, and the red curve shows the same for System R. Both systems start at the energy E_0 shown on the plot.



1. Imagine that energy flows from System B to System R. From these plots, which would be bigger in magnitude, B's decrease in entropy or R's increase?
2. Would that energy flow spontaneously happen?

Active Reading Exercise: Normalizing the Boltzmann Distribution

In the preceding Example ("Degeneracy") on p. 447, find the value of the partition function and the probability of finding the system in its ground state.

Active Reading Exercise: Small System, Big Reservoir

Consider a small system S in contact with a reservoir. The reservoir consists of 1000 atoms, each of which can have energy 0, or ϵ, or 2ϵ, etc. The entire system (S and the reservoir together) has a combined energy of 2ϵ.

1. Suppose System S is in a microstate (which we'll call M_1) with energy ϵ. How much energy is left for the reservoir? How many possible microstates are available in the reservoir?
2. Now suppose System S is in a microstate (M_2) with energy 2ϵ. How much energy is left for the reservoir? How many possible microstates are available in the reservoir?

When answering the following two questions, do *not* use the Boltzmann distribution. Instead use the fact that the combined system (S and the reservoir together) obeys the fundamental assumption of statistical mechanics.

3. Based on your answers above, can you reasonably conclude that System S is more likely to be in Microstate M_1 than Microstate M_2?
4. Based on your answers above, can you reasonably conclude that System S is more likely to have energy ϵ than energy 2ϵ?

Active Reading Exercise: The $E \sim k_B T$ Guideline

Suppose a system of particles obeys the Boltzmann distribution, meaning that, for each particle, each of its states has a probability proportional to $e^{-E/(k_B T)}$. What additional assumptions must we make to conclude that each particle's average energy will be roughly $k_B T$?

We're going to give two important answers to that question. We hope you spot at least one of them before we say it, because it's a point that we emphasized in Section 10.4. (Yes, that's a hint.)

Active Reading Exercise: Heat Capacity

Estimate the order of magnitude of the heat capacity per molecule of most objects at room temperature. *Hint: Think about what we've said about the average energy per molecule.*

Active Reading Exercise: Boltzmann Factor for Speed

A molecule of mass m is in a gas at temperature T. Each possible velocity the molecule could have is a microstate of that molecule. (We're ignoring position.)

1. What is the Boltzmann factor for a microstate with speed v?
2. At room temperature the Boltzmann factor for a nitrogen molecule to be moving at 300 m/s is about five times as large as the Boltzmann factor for it to be moving at 600 m/s, but it is *not* true that a nitrogen molecule is five times as likely to be moving at 300 m/s as it is at 600 m/s. Why not?

Active Reading Exercise: The Fermi–Dirac Distribution at Low Temperatures

Answer the first two questions below based on Equation (10.6), the Fermi–Dirac distribution.

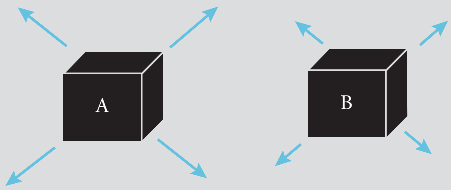
1. What is $\lim_{T \to 0^+} \overline{n}$ for $E < \mu$?
2. What is $\lim_{T \to 0^+} \overline{n}$ for $E > \mu$?
3. Based on both of your answers, describe the occupancy of a low-temperature system of fermions. Explain briefly why this answer comes as no surprise.

Active Reading Exercise: Two Blackbodies

Imagine two blackbodies of the same size, shape, and temperature sitting near each other. For simplicity we'll assume that nothing else around them is emitting significant amounts of radiation. As each one absorbs radiation emitted by the other one, energy is being transferred back and forth. That transfer counts as heat, not work.

1. Based on the definition of temperature, what can you conclude about the amount of energy transfer between the two systems?

Now suppose we were to tell you that Blackbody A emits more radiation than Blackbody B.



2. Some of the radiation emitted by Blackbody A strikes Blackbody B. What happens to that radiation?
3. Based on your answer to Part 2, does Blackbody A absorb more radiation than Blackbody B, or the other way around, or the same?
4. Your answers should have uncovered a logical contradiction. What can you therefore conclude?

Active Reading Exercise: Condensation in a Two-State System, Part 1 (compare p. 490)

Consider a system composed of N non-interacting particles at temperature T. Each individual particle has only two possible states, one with energy $E=0$ and the other with energy $E=\epsilon$.

1. In the limit $T \to 0^+$, what will the particles do? (This part requires no calculations, and the answer is the same for both distinguishable particles and bosons.)
2. Assuming the particles are distinguishable, about how many will be in the ground state if $T = \epsilon/k_B$? *Hint*: Apply the Boltzmann distribution to a single particle.
3. Now assuming the particles are indistinguishable bosons, about how many will be in the ground state if $T = \epsilon/k_B$? *Hint*: Apply the Boltzmann distribution to the system as a whole. (So, also assume the existence of an external reservoir.) The system has $N+1$ possible states, but you should find that they become irrelevant after the first two or three.

This might take you five or ten minutes – longer than most of our Active Reading Exercises – but we encourage you to take the time. If you work through all three parts, or try your best and get stuck before you read our answers, you will understand why Bose–Einstein condensation happens better than you could by just reading a derivation.

Active Reading Exercise: Condensation in a Two-State System, Part 2

For the system of distinguishable particles above, we applied the Boltzmann distribution to a single particle; for the bosons, we applied it to the whole system. Now, still supposing the existence of an external reservoir, suppose we applied the Boltzmann distribution to the entire system of distinguishable particles. How would those calculations differ from the calculation for bosons, and what effect would that difference have on the results?

Active Reading Exercise: Cohesive Energy

Imagine a large-but-finite collection of sodium atoms and chlorine atoms – not ions, regular atoms – all very far away from each other. Over time they get together, exchange a few electrons, and end up in the lattice in Figure 11.2.

There are many energetic differences between the original configuration (individual atoms) and the final configuration (salt). Equation (11.1) represents two of those differences: the attraction between oppositely charged ions is a negative contribution to the final potential energy, and the repulsion between same-charged ions is a positive contribution. List as many *other* differences as you can, classifying each one as negative or positive.

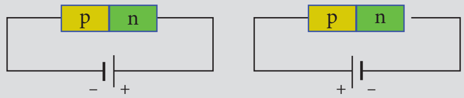
Active Reading Exercise: Available Electrons in a Crystal

Imagine a crystal whose band structure includes the one "forbidden" and two "allowed" bands portrayed in Figure 11.7 on p. 509.

1. Suppose $\epsilon_F = 2.5$ eV, so the highest-energy filled level is in the middle of the bottom (allowed) band. If an external source supplies 1/10 eV, are there electrons that can absorb it?
2. Now suppose $\epsilon_F = 3.5$ eV, meaning that the bottom band is entirely filled and the top band entirely empty. If an external source supplies 1/10 eV, are there electrons that can absorb it?

Active Reading Exercise: A p-n Junction

The figure below shows two p-n junctions connected to batteries. In which of these two arrangements will current flow through the junction?



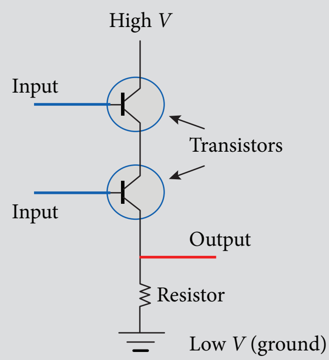
Even if you don't know the answer, write down a few thoughts about how the two configurations might differ. Remember that the difference is *not* that the n-side of the junction is negatively charged or the p-side positive; they are both electrically neutral! The difference, as we explain in the text, has to do with available charge carriers.

Active Reading Exercise: A Transistor

Suppose a positive voltage is applied to the gate in Figure 11.21, creating an electric field that points downward through the channel below it. Does that increase or decrease the current between the source and drain? Why?

Active Reading Exercise: A Logic Gate

The figure below shows a circuit with two inputs and one output. Each input is connected to the gate of one transistor. Using what we've said about transistors, is this circuit one of the logic gates listed above (AND, OR, XOR, or NAND), or something different? To answer, work out what will happen in the circuit for each possible combination of high and low voltages for the inputs. Remember that a resistor with current flowing through it has a voltage difference across it, while a resistor with no current has essentially the same voltage on both sides.



Active Reading Exercise: Magnetic Susceptibility

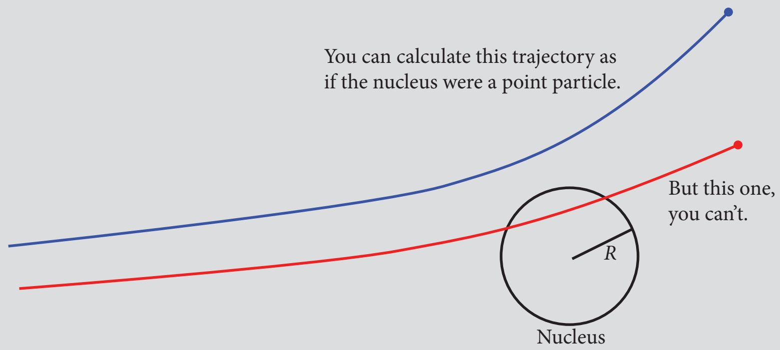
1. What is the sign of χ for a diamagnet?
2. What is the sign of χ for a paramagnet?
3. Why can't we meaningfully ask the same question about a ferromagnet?

Active Reading Exercise: The Einstein Model of Heat Capacity

1. For $k_B T < \hbar\omega$, will the average energy per oscillator be higher or lower than the equipartition theorem predicts? Why?
2. Explain how that will affect the heat capacity.

Active Reading Exercise: A Finite Size Nucleus

The Rutherford scattering formula is based on a point-like nucleus. That model works as long as the alpha particle stays outside the nucleus, because the force from a spherical charge is the same as if it were all at one point. But when the distance of closest approach is smaller than the radius of the nucleus, Rutherford's formula fails.



For trajectories such as the red one in this diagram, would Rutherford's formula predict too large or too small a deflection angle?

Active Reading Exercise: Mass Spectrometry

The centripetal acceleration that keeps the atom in Figure 12.8 moving in a circle is supplied by the magnetic force $\vec{F} = q\vec{v} \times \vec{B}$. Use that fact to calculate the mass M of the atom as a function of e, R, B, and E.

Active Reading Exercise: Beta Decay

When a neutron decays into a proton it produces other particles as well.

1. What conservation law would be violated if a neutron decayed into a proton and nothing else?
2. What other conservation laws does this process have to obey? See if you can come up with at least three.

Active Reading Exercise: Different Types of Particles

Consider two point particles. List all the properties you can think of that might distinguish these two particles, such that we would say that they are not the same type of particle. (With a bit of thought, you should be able to think of the three we list on p. 597.)

Active Reading Exercise: Building Hadrons

In the questions below, "quark content" refers to quark flavors, e.g. *uud* for a proton.

1. What's the quark content of the lightest possible baryon with charge -1?
2. What's the quark content of the lightest possible meson with charge -1?

Active Reading Exercise: The Higgs/Water Analogy

When an object is underwater, it is harder to get it moving, which makes it feel like it's more massive than it would be in air. List at least two ways in which the object's properties underwater are *not* analogous to having more than its usual mass.

Active Reading Exercise: Relativity and Bubble Chambers

Suppose a charged particle is moving through a bubble chamber at 0.99995 c.

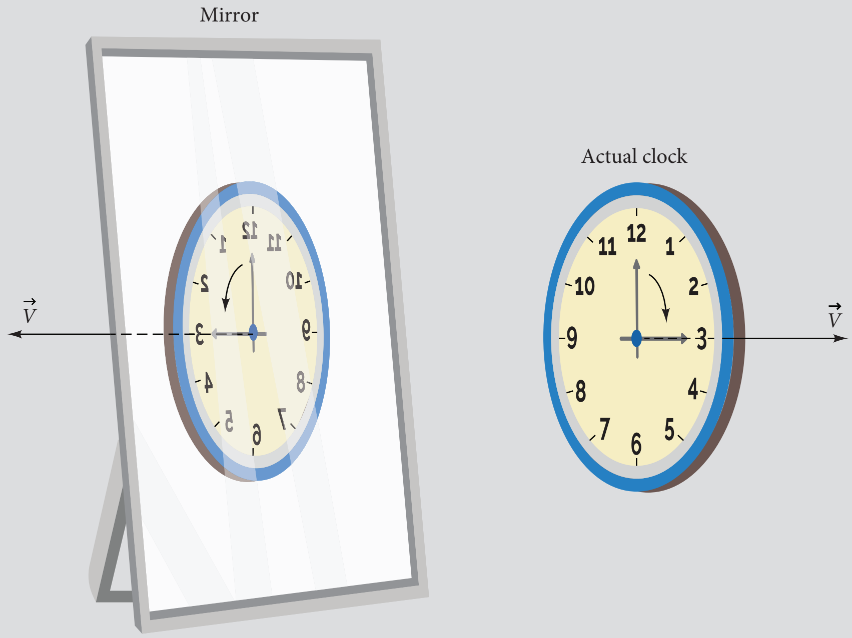
1. How long would it take for that particle to make a 10^{-6} m track, according to the lab frame?
2. How much time would elapse while it made that track, according to its own reference frame?
3. If the particle's lifetime were somewhere between your two answers, would it make a track long enough for us to detect or not?

Active Reading Exercise: Detecting a Short-Lived Particle

A Δ^+ particle at rest in the lab frame decays into a π^+ and n. You capture and measure the energies and momenta of the two outgoing particles. Write the equation for the mass of the Δ^+ particle in terms of your measured quantities. Note that these particles are moving at relativistic speeds, so you need relativistic formulas (Appendix B).

Active Reading Exercise: A Right-Handed Clock

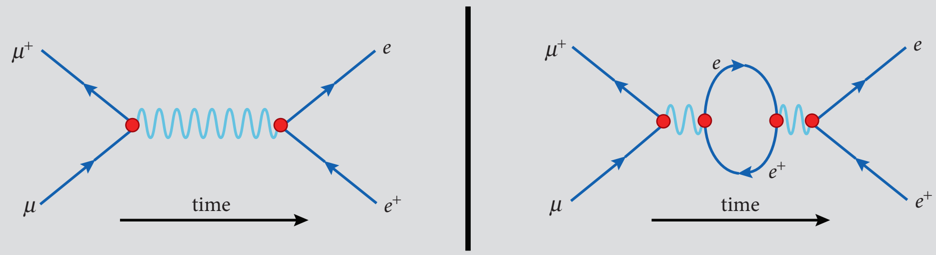
The clock in the figure is moving to the right while its hands turn clockwise around the dial. You are watching the clock, and also watching its reflection in a mirror.



1. What is the direction of the angular momentum vector for the hands of the actual clock?
2. What is the direction of the angular momentum vector for the hands of the reflected clock?
3. A “right-handed clock” has its angular momentum vector parallel to its momentum vector; a “left-handed clock” has the two vectors antiparallel. Classify both the actual clock, and the mirror clock, as right-handed or left-handed.

Active Reading Exercise: Reading Feynman Diagrams

The figure shows two different Feynman diagrams, representing two different sets of events. The result you would see in the lab – the incoming particles, and the resulting particles – is the same for both diagrams. We therefore refer to these as two different paths for the same reaction.



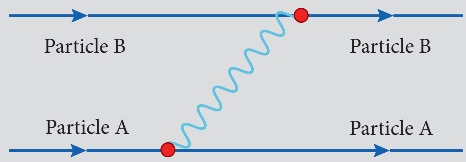
1. What particles go into this reaction? What particles come out? (Remember that time always flows from left to right; the backward arrows simply designate antimatter particles.)
2. Describe the step-by-step process represented by the diagram on the left.
3. Describe the step-by-step process represented by the more complicated diagram on the right.

Active Reading Exercise: Creating More Paths

The Active Reading Exercise on p. 635 shows two Feynman diagrams for the process of a muon and an antimuon colliding and producing an electron and a positron. Both diagrams are constructed from allowed QED vertices. Draw two other allowed Feynman diagrams for the same process (meaning the same list of initial and final particles).

Active Reading Exercise: Force as an Exchange of Particles

The Feynman diagram below contains two vertices, both of which obey the QED rule discussed above. It describes a "scattering" process, meaning that it begins and ends with the same set of particles. The left-to-right timeline tells us that the event "Particle A interacts with photon" precedes Particle B's similar event, so we can read this diagram as saying that Particle A emitted a photon that was subsequently absorbed by Particle B.



Based on classical conservation of momentum, does this interaction …

A. accelerate both particles toward each other?

B. accelerate both particles away from each other?

C. accelerate one toward the other and the other one away?

Briefly explain your answer.

Active Reading Exercise: Nucleosynthesis

Before the universe reached the one-minute mark, the ambient temperature was too hot for protons and neutrons to bond. After the temperature dropped sufficiently, nuclei began to form.

The strong force was just as strong before one minute as it was after. So why was nucleosynthesis impossible until the temperature went down enough?

Active Reading Exercise: Static or Expanding?

1. Imagine you look outside for two seconds. During that two seconds you watch a ball in midair. The only force acting on this ball is the Earth's gravitational pull. (The following questions are as trivial as they sound.)
   (a) Is it possible for the ball to be moving down the whole time you are watching it?
   (b) Is it possible for the ball to moving up the whole time you are watching it?
   (c) Is it possible for the ball to be at rest the whole time you are watching it?
2. Now imagine you spend a billion years looking at an entire universe that consists of 100 galaxies floating in space. There's nothing else in this universe. Once again, the only relevant force is gravity: in this case, the gravitational attraction between all the galaxies.
   (a) Is it possible for the galaxies to be moving toward each other the whole time you are watching them?
   (b) Is it possible for the galaxies to be moving away from each other the whole time you are watching them?
   (c) Is it possible for the galaxies to all be at rest the whole time you are watching them?
   (d) Would any of your answers be different if there were infinitely many galaxies spread through an infinite amount of space?

Active Reading Exercise: Critical Density

For a universe expanding with Hubble parameter H, the "critical density" ρ_c is the density that universe would have to have in order for it to be flat ($k = 0$).

1. Using the first Friedmann equation, write a formula for the critical density ρ_c as a function of H.
2. In a closed universe ($k > 0$), is the actual density ρ bigger than, equal to, or smaller than ρ_c? How do you know?

Active Reading Exercise: Matter, Radiation, and Energy Density

Suppose a region of space expands, and the scale factor $a(t)$ eventually reaches double its original value.

1. Remember that $a(t)$ is a unitless measure of the distance between objects. So when a doubles, what happens to the volume of this region?
2. Matter obeys the relation $\rho \propto a^{-3}$, where ρ is the energy density (measured for instance in $\mathrm{J/m^3}$). So when a doubles, the energy density of matter goes down by a factor of 8. Based on that and the volume change you just calculated, what happens to the total energy of the matter in this region?
3. Radiation obeys the relation $\rho \propto a^{-4}$. Based on that and the volume change you just calculated, what happens to the total radiation energy in this region?

Active Reading Exercise: The Fate of the Universe

The Hubble parameter $H = \dot{a}/a$ describes how fast the universe is expanding. Equation (14.1) expresses the Hubble parameter squared as the difference between two terms: one term based on the density ρ (which is itself a function of time), and the other on the curvature k (a constant).

As the universe expands, ρ decreases (which causes the first term to decrease), while a increases (which causes the second term to *also* decrease).

1. In a matter-dominated universe, $\rho \propto a^{-3}$. If a doubles, the ρ-based term in Equation (14.1) will drop by what factor? The k-based term in Equation (14.1) will drop by what factor? Which one is decreasing faster?
2. Assuming $k > 0$ (still for a matter-dominated universe), what does your answer to Part 1 imply about what's going to eventually happen to H? What would that mean for the expansion of the universe?

Active Reading Exercise: Dark Energy

Consider a universe dominated by a type of energy density ρ that is constant as the universe expands. (That's constant energy *density*, not total energy.) For a flat universe, the first Friedmann equation then says:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^2}\rho \quad \textit{is constant}.$$

You can solve that as is, but the math looks simpler if we use H for $\dot{a}/a$ as usual:

$$\left(\frac{\dot{a}}{a}\right)^2 = H^2 \quad \textit{is still constant}.$$

1. Solve this equation for $a(t)$. Your solution will have the constant H in it.
2. Looking at your solution to Part 1, would you see a distant galaxy's motion away from you speeding up or slowing down? How can you tell?

Active Reading Exercise: Two Particles with Different Forces

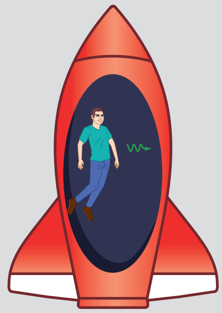
Nothing in this exercise requires special relativity per se. However, you do need to be familiar with "spacetime diagrams," as described in Section 2.1.

A proton moves along the x-axis. At time $t = 0$ the proton is at position $x = 0$ with a positive velocity. We aren't going to put a number to that velocity, so your goal in this exercise is to get the overall shapes right, not the numbers.

1. Draw a spacetime diagram that represents the proton's journey if no forces act upon it.
2. Starting over on a blank graph, draw a different spacetime diagram that represents the proton's journey if it is immersed in a constant electric field pointing in the negative x-direction. (Perhaps the proton is inside a capacitor.)
3. On a third graph, draw a third spacetime diagram that represents the proton's journey if it is immersed in a constant *gravitational* field pointing in the negative x-direction. (Perhaps the proton is on Earth, and the x direction is "up.")
4. Now replace the proton with an electron: identical-but-opposite charge, much smaller mass, same initial position and velocity. On each of the three spacetime diagrams you created for the proton, add a second path for the electron. Our particular interest here is whether, and how, the electron's path differs from the proton's path in each scenario.

You should create three spacetime diagrams, with two curves each, before you continue reading!

Active Reading Exercise: A Flashlight on a Falling Ship



You and Al are back in your sealed space ship, and you have come up with a new experiment to try. You fire a single photon across the ship and measure the spot where that photon hits the far wall.

Question: Will the photon hit directly across from where you fired it, or slightly below that spot, or slightly above that spot?

Remember that one of our two possible scenarios was your ship floating in space, with no gravity or acceleration. In that case the answer to our question is obvious: the photon will hit directly across from where you fired it.

But where does the photon hit in the other scenario, with the ship in free fall toward the Earth?

1. Answer this question based on a classical description of gravity. *Hint:* Photons are massless.
2. Now answer the same question based on general relativity.

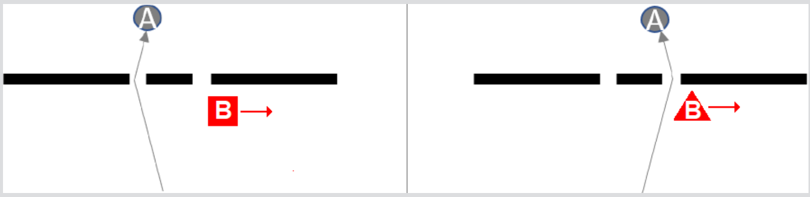
Active Reading Exercise: Light Beams in an Expanding Universe

Consider a spacetime with the metric $ds^2 = -(a(t))^2 dx^2 + dt^2$. Using the fact that a light beam always moves along a path with $ds = 0$, calculate the speed dx/dt of a light beam in this spacetime as a function of time. Using your result, sketch a spacetime diagram with a light cone whose vertex is at the origin. *Hint*: It will not look like a cone.

Active Reading Exercise: Measurement by Shape

You send Particle A through a double slit. Particle B, passing nearby, has a property that we're calling "squareness."

- If Particle A passes through the left-hand slit, then Particle B remains square.
- But if Particle A passes through the right-hand slit, then Particle B changes to a different property that we're calling "triangleness."



To introduce an important quantum mechanical word, the states of our two particles are now "entangled." There is a 50% probability of finding this system in the state "A went through the left slit and B is square," and a 50% chance of finding "A right slit, B triangle." But there is zero chance of finding the "A left slit, B triangle" state. Each member of an entangled pair of particles is in a superposition of states, but it's a special sort of superposition where a measurement of one of them determines the properties of both.

Particle B now goes flying off in a different direction. But Particle A continues on to the back screen of the double-slit, where you will measure its position. You will then repeat the experiment many times, measuring the positions of many "Particle A"s.

1. After you repeat this experiment many times, is there an interference pattern on the back screen of the double-slit experiment?
2. In an alternative version of the experiment, B flies into an apparatus that changes it into a circle. You never tested it to determine whether it was a square or a triangle, and it is now impossible to recover that information. Is there still an interference pattern on the back screen?

Jot down your answers to those questions before reading on!

Active Reading Exercise: Measurement by Polarization

Imagine that you are doing Walborn's experiment, as described above.

1. Suppose you measure that Photon A reaches the back screen right-polarized, and Photon B is z-polarized. Can you conclude that Photon A went through the left-hand slit, or can you conclude that it went through the right-hand slit, or do you not have enough information to determine what slit it went through?
2. Now suppose you send many photons through this system, but you never measure their polarizations, so each system remains in a superposition of the four states shown in Figure 3. Would the A photons create an interference pattern? Why or why not?
3. Finally, suppose you measured each Photon B to determine if it was y or z-polarized. Would that measurement change your answer to Part 2? Explain.

Active Reading Exercise: The Sum of Two Cosines

1. What is the period of the function $\cos(x)$? What is the period of $\cos(2x)$?

Now consider the following combination of those two functions:

$$f_1(x) = \frac{1}{2}\cos(x) + \frac{1}{2}\cos(2x).$$

Answer both of the following questions *without* using a computer or calculator to graph $f_1(x)$ for you—but then, of course, feel free to use such a graph to check your answers.

2. At $x = 0$ our function reaches its maximum value, $f_1(0) = 1$, because of perfect constructive interference of the two cosines. What are all the other x-values at which this function attains that maximum?
3. What are all the x-values at which this function reaches perfect *destructive* interference, i.e. $\cos(x) = -\cos(2x)$ (which implies $f_1 = 0$)?

Active Reading Exercise: Lots of Cosines

Our next function adds many more cosines, but still a finite number.

$$f_2(x) = \frac{1}{10}\cos(x) + \frac{1}{10}\cos(2x) + \frac{1}{10}\cos(3x) + \cdots + \frac{1}{10}\cos(10x)$$

1. List the periods of all ten cosine functions that make up $f_2(x)$. (This isn't as onerous as it sounds: once you see the pattern, the whole list will take less than thirty seconds.)

2. What are all the x-values at which this function reaches perfect constructive interference, i.e. $f_2(x) = 1$?

3. When x is not at or very near the values you listed in Part 2, $f_2(x)$ stays very close to zero. Based on the behavior of its component cosine functions, explain why.

4. Extrapolating from what you've seen so far, draw a rough sketch of what you think the following function would look like.

$$f_3(x) = \frac{1}{30}\cos(x) + \frac{1}{30}\cos(2x) + \frac{1}{30}\cos(3x) + \cdots + \frac{1}{30}\cos(30x)$$

Active Reading Exercise: Half-Integer Frequencies

The previous Active Reading Exercise introduced the function $f_3(x)$, the sum of thirty cosine functions. Figure 4 on p. 5 of the Wave Packets chapter shows the graph of $f_3(x)$.

The function f_4 below is *also* the sum of thirty cosine functions, but their frequencies are half the frequencies of the corresponding cosines in f_3:

$$f_4(x) = \frac{1}{30}\cos\left(\frac{x}{2}\right) + \frac{1}{30}\cos(x) + \frac{1}{30}\cos\left(\frac{3x}{2}\right) + \frac{1}{30}\cos(2x) + \cdots + \frac{1}{30}\cos(15x).$$

How does the graph of f_4 differ from the graph of f_3?

Active Reading Exercise: The Sum of Two Complex Exponentials

1. What is the period of the function e^{ix}? What is the period of e^{2ix}?

Now consider the following combination of those two functions:

$$f_5(x) = \frac{1}{2}e^{ix} + \frac{1}{2}e^{2ix}.$$

2. At $x = 0$ our function has its maximum modulus, $|f_5(0)| = 1$, because of perfect constructive interference. What are all the other x-values at which this function has modulus 1?
3. What is the first x-value at which this function reaches perfect *destructive* interference, i.e. $e^{ix} = -e^{2ix}$ (which implies $f_5(x) = 0$)?